Unsaturated Hydraulic Conductivity Measurements with Centrifuges: A Review

E. H. van den Berg, E. Perfect,* C. Tu, P. S. K. Knappett, T. P. Leao, and R. W. Donat

Conducting drainage or imbibition experiments in a centrifugal force field has long been recognized as a valid and efficient way to determine capillary pressure–saturation and relative permeability–saturation relationships. Experiments involving multiphase flow of immiscible fluids in porous media are sped up because the centrifugal acceleration is many times greater than Earth’s gravitational acceleration. In addition, the fact that centrifugal force is a body force and that experiments can be performed under well-controlled conditions are considered advantages over other techniques. During the past few decades, transient-flow centrifuge methods have been developed and applied in petroleum geosciences, while in the soil and environmental sciences the focus has been on steady-state centrifuge methods. To inform both groups of each others’ work, the different instrumental approaches used are described. The theoretical background for modeling multiphase fluid flow of immiscible fluids through porous media in a centrifugal force field is then reviewed. This background forms the basis for understanding analytical and numerical formulations to interpret both steady-state and transient-flow centrifuge experiments. Research on the effects of compaction, menisci deformation, boundary conditions, corrections for early-time production data, the selection of measurable variables, and nonuniqueness of the data interpretation is discussed. A numerical example application of the transient-flow centrifuge method is presented. Finally, the major conclusions that can be drawn from the literature are discussed and potential areas for future research are identified.

**Abbreviations**: IFC, internal flow control; UFA, unsaturated flow apparatus.

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GEOSCIENTISTS are often faced with making quantitative predictions of the migration of multiple fluids through the subsurface. Software packages for simulating multiphase fluid flow through porous media are well developed (e.g., ECLIPSE Schlumberger reservoir simulator, HYDRUS-1D and 2D [Šimůnek et al., 1998, 1999], and STOMP [White and Oostrom, 2000]) and find wide applications in both research and industry. Among the physical properties of the porous medium that are required as model inputs, the capillary pressure and unsaturated hydraulic conductivity (or relative permeability) vs. fluid saturation functions are often considered the most critical (Christiansen, 2001). Accurate and efficient measurements of these functions are therefore of considerable importance for ensuring accurate simulations.

The advantages of measuring the capillary pressure and relative permeability functions on core samples using centrifuges have been recognized from as early as the beginning of the 20th century (Briggs and McLane, 1907; Gardner, 1937). The fact that the centrifugal acceleration is many times greater than Earth’s gravitational acceleration and that it acts equally on all fluids throughout the sample (which accelerates the adaptation to changes in boundary conditions) is seen as an attractive way to speed up multiphase fluid flow experiments. Besides this advantage, the flow and transport processes take place under more ideal and controlled conditions than experiments conducted under normal gravity, leading to more accurate measurements that extend across a wider range of saturations (Nimmo, 1990; Nakajima and Stadler, 2006).

Conventional permeameter techniques for measuring the intrinsic permeability, \(k\), under normal gravity conditions have also been applied to a centrifugal field. A clear distinction is made between experiments in which water is supplied from an internal or an external source, and if the flow experiment is conducted under constant- or falling-head conditions (Nimmo et al., 2002). The relevant analytical equations describing saturated flow under these different combinations of experimental conditions were presented in Nimmo and Mello (1991) and Nimmo et al. (2002).

Centrifuge techniques for measuring the relative permeability function can be divided into steady- and transient-state methods. In steady-state centrifuge experiments, the sample and fluids are subject to a time-invariant centrifugal acceleration. Pressure and flow conditions at the inlet end face (the sample face closest to the center of rotation) are set so that steady-state flow conditions develop for the wetting fluid with time. Two steady-state methods, referred to by Nimmo et al. (2002) as internal flow control (IFC) and unsaturated flow apparatus (UFA), have been used for multiphase flow applications.
A steady-state centrifuge experiment typically consists of centrifuging samples at different angular velocities or flow rates, reaching steady-state unsaturated flow conditions for each speed and determining the average water content. Using measured or calculated fluid pressures and measured flow rates, the relative permeability that corresponds to the measured average saturation is calculated. The steady-state approach has been widely used for measuring flow and transport parameters under partially saturated conditions (Nimmo et al., 1987, 1994, 2002; Nimmo and Mello, 1991; Conca and Wright, 1992, 1998; Conca, 1993; Conca et al., 1997, 1998; ASTM, 2000; Seaman et al., 2002; Basha and Mina, 1999; Caputo and Nimmo, 2005; Wright et al., 1994). Both the IFC and UFA methods ignore the gradient in capillary pressure along the length of the sample. This can lead to significant bias in the estimation of relative permeability under high-capillary-pressure conditions. Nimmo et al. (1987) discussed various approaches for evaluating and correcting for capillary pressure gradients.

Studying the large body of petrophysical literature on the relative permeability function, it is evident that transient-flow centrifugation is the standard method; no applications of steady-state centrifuge methods were encountered in this field. The transient experiment always consists of a period of time in which the centrifuge rotor is accelerating to the specified constant angular velocity and a period in which the angular velocity is constant. Because of capillary forces, there will be no production until the capillary pressure generated by the centrifugal acceleration exceeds the air-entry pressure. Therefore, the angular velocity at the time production starts is used to calculate the air-entry pressure. Similar to a pressure cell experiment under normal gravity conditions, the step change between initial saturation and saturation at air-entry pressure (also called the saturation or mobility shock front) travels from the inlet to the outlet end face of the core. The slope of the shock front production has been used to determine the intrinsic permeability of the sample (Hagoort, 1980). After passing of the shock front, the changes in fluid saturation within and fluid production outside the sample are the result of an imbalance between the transient capillary pressure profile and the constant centrifugal acceleration profile. Technical improvements during the past few decades have made it possible to automatically measure fluid outflow from the core during centrifugation (O’Meara and Lease, 1983; Hirasaki et al., 1992, 1995). Interpretation methods for single-rate transient-flow centrifuge experiments include both analytical solutions (Hagoort, 1980; van Spronsen, 1982) and numerical approximations (O’Meara and Crump, 1985; Hirasaki et al., 1992; Firoozabadi and Aziz, 1991) of the multiphase fluid flow problem. Multi-rate transient-flow centrifuge experiments have been performed and interpreted with inverse numerical simulation software for the simultaneous estimation of the capillary pressure and relative permeability functions (O’Meara and Crump, 1985; Chardaire-Riviere et al., 1992; Al-Omair and Christiansen, 2005).

There are a limited number of examples in which transient-flow centrifuge methods have been applied to vadose zone problems (Alemi et al., 1976; Nimmo, 1990; Šimůnek and Nimmo, 2005; Nakajima and Stadler, 2006). Alemi et al. (1976) presented analytical equations for calculating production through time. With the production through time measured and initial and final moisture contents known, a single value of the end-point permeability for a given centrifugal speed is determined. In an effort to test the validity of Richards’ equation under low moisture content conditions, Nimmo (1990) measured the transient moisture content conditions at several locations along the length of centrifuged cores. Using the well-characterized Oakley sand, changes in moisture content through time were simulated for a single-rate experiment with a numerical model introduced by Bear et al. (1984). The correspondence between simulated and observed point moisture contents confirmed the validity of Richards’ equation under low moisture content conditions. Following up on the numerical modeling, Šimůnek and Nimmo (2005) introduced a modified version of the HYDRUS-1D software package for simulating transient water flow in a multi-rate centrifuge experiment. They used the inverse simulation option of HYDRUS-1D to determine best-fit parameter values and associated uncertainties. Interestingly, they mentioned that the discharge rate of the fluid, as well as the concentration of chemical compounds produced from the sample, can also be used in parameter estimation. Nakajima and Stadler (2006) were the first soil or environmental scientists to inversely estimate soil hydraulic parameters using measured fluid production as calibration data.

With the recent increase of interest in transient-flow centrifuge methods (Šimůnek and Nimmo, 2005; Nakajima and Stadler, 2006), many soil and environmental scientists may not be fully aware of the valuable developments, insights, and experiences provided by the petrophysics community. Therefore, this review begins by presenting the different instrumental approaches used. The theoretical background for modeling multiphase fluid flow through porous media in a centrifugal force field is then reviewed. This background forms the basis for understanding commonly used analytical and numerical formulations to interpret steady- and transient-flow centrifuge experiments. Research on the effects of compaction, meniscus deformation, boundary conditions, corrections for early-time production data, the selection of measurable variables, and nonuniqueness of the data interpretation is discussed. An illustration of analytical and numerical inversion techniques applied to a single-rate, transient-flow centrifuge experiment is then presented. Finally, the major conclusions that can be drawn from the literature are discussed and potential research areas for the future are identified.

**Instrumentation**

Starting with the steady-state UFA system, three differently sized and specially adapted centrifuge rotors can be mounted in a standard J6-M Beckman (Beckman Coulter, Fullerton, CA) ultracentrifuge (Conca and Wright, 1998). The most commonly used UFA centrifuge rotor has a maximum arm length (radial distance from the center of rotation to the platform on which the sample rests) of 11.5 mm and houses two fixed centrifuge buckets that can contain samples 49.0 mm long and 30.0 to 40.0 mm in diameter (Fig. 1). The centrifuge rotors are modified so that fluids can be supplied by means of a rotating seal assembly, through external infusion or syringe pumps, to the inlet end faces of the core samples while the rotor is spinning. The fluid that passes through the samples (effluent) is collected in receiving cups that are located at the outflow ends of the cores. The effluent is then used to determine the flow rate or concentration of the fluid. The cups are manufactured of transparent plastic and are volumetrically calibrated. A strobe light and eyepiece allow manual
measurement of the cumulative outflow of fluids during the experiment. Although Beckman has discontinued production of this centrifuge model, modifications are still being made by UFA Ventures, Richland, WA.

The steady-state IFC method (Nimmo et al., 1987) consists of a self-contained cell that houses a sequence of water storage reservoirs and flow control devices at the inflow and outflow end faces of the core. The experimental setup is illustrated in Nimmo et al. (2002). The flow of water into the core is controlled by ceramic plates, the water flow rate depending on the hydraulic properties of the ceramic plates and centrifuge run conditions. Nimmo et al. (1994) used a 1-L swinging bucket ultracentrifuge with an arm length of 210 mm. Steady-state fluid production is approached after prolonged centrifugation and the unsaturated hydraulic conductivity at the specific moisture content can be calculated. Replacing the ceramic plates with more (or less) permeable ones allows similar measurements at higher (or lower) water contents.

As mentioned above, the petrophysics community has focused on transient-flow centrifuge methods for determining the relative permeability function. The transient-flow conditions within the sample are derived by recording the outflow rate through time during an interval of constant angular velocity. Traditionally, the flow rate was determined by using transparent, volume-calibrated receiving cups and tracking the movement of the wetting–nonwetting interface using a strobe light and optical eyepiece, as is done with the UFA. To facilitate more precise measurements, however, petrophysical researchers investigated the automated recording and detection of the interface (O’Meara and Lease, 1983; Hirasaki et al., 1992). These developments resulted in a state-of-the-art ultracentrifuge (Coretest Systems, Morgan Hill, CA). Their URC-628 Ultra-Rock centrifuge consists of two integral parts: a Beckman L-8 ultracentrifuge and an automated data acquisition system (Fig. 2). The rotor and buckets of the Beckman L-8 are very similar in design and dimensions to those of the Beckman J6-M ultracentrifuge. The centrifuge has an arm length of 86.0 mm and can carry three samples with maximum length and diameter of 57.80 and 39.65 mm, respectively. The main difference between the J6-M and the L-8 is that the centrifuge rotor of the latter is placed in a vacuum chamber. This allows the rotor to reach angular velocities of up to 1727.9 rad s⁻¹ (16,500 rpm), compared with only 261.8 rad s⁻¹ (2500 rpm) for the J6-M centrifuge. The sample holders and receiving buckets are hermetically sealed from the vacuum so that the displacement experiments take place under atmospheric conditions. An automated data acquisition system monitors the location of the wetting–nonwetting interface in the receiving cup by illumination of transparent receiving cups with a high-intensity linear light source. Images consisting of transmitted light intensity vs. distance are acquired at regular time intervals with a digital line scan camera. The light intensity signature of the interface between the wetting and nonwetting fluid is enhanced by an opaque material floating on the wetting fluid. In this way, the interface is easily recognized by image analysis software and converted to a high-resolution and accurate data set of cumulative production through time.

Geotechnical centrifuges have been used to measure the hydraulic properties of larger samples. Examples are the geocentrifuges at the Department of Civil Engineering and Engineering Mechanics at Columbia University (www.civil.columbia.edu/ling/centrifuge; verified 6 Apr. 2009), and at the Idaho National Laboratory (INL, www.inl.gov/centrifuge; verified 6 Apr. 2009). Instrumentation, traveling onboard in the centrifuge bucket, provides in-flight manipulation of experiments and data collection. Nakajima and Stadler (2006) used the INL geocentrifuge to perform transient, single-step outflow experiments on columns with a diameter of 102 mm and a length of 432 mm packed with fine Ottawa sand. The samples were instrumented with a number of miniature tensiometers at several positions along the column and

![Fig. 1. Cross-section, perpendicular to the plane of rotation, of the unsaturated flow apparatus (UFA) rotor including the two sample and bucket assemblies (image adapted from Nimmo et al., 2002).](image1)

![Fig. 2. Schematic diagram of the Ultra-Rock centrifuge (left) and automated data acquisition system (right). Diagrams are adapted from J.A. Rohan.](image2)
an outflow measuring device. A remotely operated valve was used to start the outflow experiment only after the rotor had reached the programmed angular velocity.

**Basic Theory**

Similar to kinetic energy in linear motion, the energy of an object rotating about a fixed center of rotation is a function of tangential velocity, radial distance from the center, and mass. The total energy per unit of volume of a rotating object, $E_R$, is defined as

$$E_R = \frac{1}{2} \rho r^2 \omega^2$$  

where $\rho$ is the density (kg m$^{-3}$), $r$ is the radial distance (m), and $\omega$ is the constant angular velocity (rad s$^{-1}$). The quantity in parentheses is called the moment of inertia, $I$, which is calculated differently for each object shape or configuration of objects (Alonso and Finn, 1967). Although this rotational energy has the form of kinetic energy, it can be seen as a form of potential energy driving fluid flow through a porous medium in a centrifugal field.

The flow of multiple fluids through a porous medium in a centrifugal force field is proportional to the gradient in pressure potential energy ($E_p$) and the potential energy caused by uniform circular motion ($E_R$). The resultant of these two potential energies is given by

$$\text{grad} \Phi = \text{grad} \left( P - \frac{1}{2} \rho \omega^2 r^2 \right)$$  

where $\Phi$ is the total pressure potential (kg m$^{-1}$ s$^{-2}$) and $P$ is the fluid pressure (kg m$^{-1}$ s$^{-2}$). Darcy’s law for steady-state flow of wetting fluids in a centrifugal force field can be written as

$$u_w = -k_w k A / \mu w \text{grad} \left( P_w - \frac{1}{2} \rho_w \omega^2 r^2 \right)$$  

where $k$ is the isotropic intrinsic permeability (m$^2$), $k_w$ is the relative permeability (dimensionless), $\mu$ is the dynamic viscosity (kg m$^{-1}$ s$^{-1}$), $A$ is the cross-sectional area (m$^2$), and $u$ is the volumetric flux (m$^3$ s$^{-1}$), with the subscript $w$ signifying the wetting fluid.

An interesting finding by Nimmo (1990) is that Darcian flow conditions appear to hold under extreme centrifuge conditions. Singh and Kuriyan (2002) evaluated the validity of Darcy’s law for their centrifuge experiments by calculating the Reynolds number. They concluded that water flow takes place under Darcian conditions even at high centrifuge accelerations (up to $a_c = 1226.3$ m s$^{-2}$ or $\sim 125 \times g$).

The continuity equation for homogeneous, incompressible wetting fluids in a nondeformable porous medium is given by

$$\frac{\partial P_w u_w}{\partial r} + \frac{\partial (P_w S_w \phi A)}{\partial t} = 0$$  

where $\phi$ is the porosity (dimensionless), $t$ is time (s), and $S_w$ is the fraction of pore space occupied by the wetting fluid (dimensionless). For a multiphase fluid flow experiment, the individual liquid saturations should sum to unity.

Capillary pressure relates to the difference in pressure across the interface between the wetting and nonwetting fluids. Based on the Young–Laplace equation, the pressure difference between adjacent fluids, $P_c$ (kg m$^{-1}$ s$^{-2}$), is related to the principal radii of curvature of the shared interface and the interfacial tension, $\sigma$ (kg s$^{-2}$), in the following way:

$$P_c = P_{nw} - P_w = \frac{2\sigma \cos \Theta}{\langle R \rangle}$$  

where $\langle R \rangle$ is the mean radius of curvature (m) and $\Theta$ is the contact angle (rad). Equations [3–5] represent the basic theory for describing flow of multiple fluids through a porous medium in a centrifugal force field.

**Data Interpretation**

**Steady-State Unsaturated Flow in a Centrifugal Field**

Figure 3 shows the experimental setup for measuring steady-state unsaturated flow in a centrifugal field. This setup can be modeled by rearrangement of the terms in Darcy’s equation, Eq. [3], and applying the gradient operator to each potential component, resulting in the following expression:

$$\frac{u_w}{k_w A / \mu_w} = \frac{\rho_w \omega^2 r}{dP_w / dr}$$  

Equation [6] shows the clear relationship between the gradient of fluid pressure potential and the centrifugal force: at hydrostatic equilibrium ($u_w = 0$), the gradient of fluid pressure potential is equal and opposite in sign to the centrifugal force. For steady-state flow, this implies that, when the centrifugal force is many times greater than the gradient in fluid pressure potential, the last term in Eq. [6] can be neglected. The flow equation then simplifies to

$$u_w \sim \frac{k_w (P_w / k A) / \mu_w}{P_w} \rho_w \omega^2 r$$  

By measuring $u_w$ and the average fluid pressure potential ($P_w$) or saturation ($S$), a point on the relative permeability curve is obtained. Repeating these measurements at different flow rates, $u_w$, the entire relative permeability function can be constructed.

**Analytical Model for Transient, Unsaturated Flow in a Centrifugal Field**

The experimental and theoretical work of Hagoort (1980) forms the basis of a method for estimating the relative permeability function from the transient production of a wetting fluid in the porous plate...
a single-rate centrifuge experiment (Fig. 4). The porous medium is initially saturated with a wetting fluid. The fluid production is measured at regular intervals through time and interpreted using implicit or explicit methods.

In the derivation of the equations of Hagoort (1980), it is assumed that the mobility of the invading nonwetting fluid is so high that the gradient of total potential for that fluid can be neglected. Thus, the total potential of the nonwetting invading fluid is spatially uniform and temporally constant, which can be written as

$$\Phi_{nw} = P_{nw} - 0.5 \rho_{nw} \omega^2 r^2 = C$$ \[8\]

In experiments that take place under atmospheric conditions, the constant $C$ corresponds to air pressure or a gauge pressure of zero. By combining Eq. [8] with the definition of capillary pressure in Eq. [5], the total potential of the wetting fluid can be written as follows:

$$\Phi_w = P_w - 0.5 \rho_w \omega^2 r^2 = C + 0.5 \Delta \rho \omega^2 r^2 + P_c$$ \[9\]

where $\Delta \rho$ is the density difference between the wetting and nonwetting fluids ($= \rho_w - \rho_{nw}$).

Adjusting Eq. [3] from volumetric flux ($L^3 T^{-1}$) to flux density ($q_w = u_w/A$, $L T^{-1}$), using the average centrifugal acceleration in the sample, $a_{c, \text{avg}} = 0.5 \omega^2 (r_i + r_o)$, where $r_i$ and $r_o$ are the radial distances to the inlet and outlet end faces of the core, and inserting the total potential for the wetting fluid, Eq. [9], the following expression for the Darcy flux is obtained:

$$q_w = \frac{k_{rw}}{\mu_w} \left( \frac{\Delta \rho a_{c, \text{avg}}}{\mu_w} + \frac{\partial P_c}{\partial r} \right)$$ \[10\]

Assuming that the capillary pressure gradient is small (and thus negligible) when compared with the density difference or average acceleration, Eq. [10] reduces to

$$q_w \approx \frac{k_{rw}}{\mu_w} (\Delta \rho a_{c, \text{avg}})$$ \[11\]

which is similar to Eq. [7]. Note that $k_{rw}$ is a function of saturation. Inserting the above equation into the mass balance equation of the wetting fluid, Eq. [4], and assuming the density of the wetting fluid is constant yields the following expression:

$$\frac{\partial}{\partial r} \left( \frac{\Delta \rho a_{c, \text{avg}} k_{rw}}{\mu_w} \right) + \frac{\partial S_w}{\partial t} = 0$$

or

$$\left( \frac{\Delta \rho a_{c, \text{avg}} k_{rw}}{\mu_w} \right) \frac{\partial k_{rw}}{\partial r} + \frac{\partial S_w}{\partial t} = 0$$ \[12\]

The term $\partial k_{rw}/\partial r$ can be rewritten as $(\partial k_{rw}/\partial S_w)(\partial S_w/\partial r)$ or $k_{rw}^\prime (\partial S_w/\partial r)$ with $k_{rw}^\prime = (\partial k_{rw}/\partial S_w)$.

An analytical solution to the above equation was provided by Hagoort (1980) and Christiansen (2001) and consists of two ordinary differential equations, the so-called Buckley–Leverett solutions, which can be found using the method of characteristics (Bear, 1972, p. 468–472). The initial and boundary conditions are as follows:

For $t \leq 0$, $r_i \leq r \geq r_o$, $S_{nw} = 0$

For $t > 0$, $r_i$, $u_w = 0$, and $u \neq 0$

Rewritten for Eq. [12], the second Buckley–Leverett solution is given by (Christiansen, 2001)

$$r_{Sw} = \frac{\left( \Delta \rho a_{c, \text{avg}} k_{rw}^\prime \right)}{\mu_w \phi} \int_{r_i}^r S_w(r) \, dr$$ \[13\]

where $r_{Sw}$ is the location of a specific water saturation, $S_w$, at time $t$. The location of a specific $S_w$ at different times, as well as the saturation profile along the length of the sample at a single time, can be calculated. Note that $S_w$ should lie between the residual saturation of the wetting fluid, $S_{w, r}$, and $1 - S_{nwc}$, where $S_{nwc}$ is the critical saturation at which the nonwetting fluid is fully connected. The model assumes that, during an initial period, a saturation shock front migrates from the inlet to the outlet end face of the sample, thereby establishing fully connected pathways for the wetting and nonwetting fluids. An important consequence is that the saturation of the wetting fluid at the outflow end face is reduced after the saturation shock has passed through the sample.

By integrating Eq. [13] in the spatial domain, the fluid production during an increment of time can be calculated. When the saturation profile along the length of the sample at a certain time $t$ is known, the cumulative production of wetting fluid, $Q_w$, up to that time can be written as one minus the average saturation, or

$$Q_w = 1 \frac{1}{L} \int_{S_{hw}}^{S_{nw}} S_w(r) \, dr$$ \[14\]

with $Q_w$ in pore volumes (dimensionless) and $L$ the length of the core sample (m). In this expression, it is assumed that the average saturation $\langle S_w \rangle = 1$ at $t = 0$ and consequently $Q_w = 0$. It is important to note that Eq. [14] is valid both before and after the saturation shock has passed through the sample. For the period before the saturation shock has passed, the integral
The sequence of Equations [16] and [18] are similar to Eq. [11] and [16], respectively. By optimizing the power exponent, Christiansen (2001) obtained the following expression relating saturation of the wetting fluid at the outlet end face to the slope of the production curve:

\[ Q_w = 1 - S_{w,ro} + \frac{\Delta \rho \mu_{c,avg} k t}{\phi \mu L} \int_0^t d k_{w,t} \]  

with \( S_{w,ro} \) the saturation of the wetting fluid at the outlet end face of the core (\( r_r \)). This equation can be simplified after realizing that \( u_w = 0 \) at \( r_i \). A way to make sure that \( u_w = 0 \) at \( r_i \) is to set \( k_{w,t} \) equal to zero at the inlet end face of the core. Also, \( S_w = S_{w,t} \) at this location for \( t > 0 \). With these simplifications, Eq. [15] can be rewritten as

\[ Q_w = 1 - S_{w,ro} + \frac{\Delta \rho \mu_{c,avg} k t}{\phi \mu L} k_{w,ro} \]  

Differentiating Eq. [16] with respect to time yields

\[ k_{w,ro} = \frac{\mu_w \phi L}{\Delta \rho \mu_{c,avg} k} \frac{dQ_w}{dt} \]  

Combining Eq. [16] and [17], Christiansen (2001) obtained the following expression relating saturation of the wetting fluid at the outflow end face to the slope of the production curve:

\[ S_{w,ro} = 1 - Q_w + t \frac{dQ_w}{dt} \]  

Equations [16] and [18] are similar to Eq. [11] and [16], respectively, of Hagoort (1980).

It is interesting to trace the different approaches that Hagoort (1980) and Christiansen (2001) used to adapt the above model for the interpretation of production data. Hagoort (1980) inserted the Brooks and Corey (1964) relative permeability function into Eq. [16]. By optimizing the power exponent, \( \varepsilon \), in this function, the calculated production curve can be fitted to the measured production. In contrast, Christiansen (2001) used a three-point numerical scheme for differentiating unequally spaced temporal data. The gradient of production with respect to time, \( dQ_w/dt \), thus obtained is inserted directly into Eq. [17] and [18], resulting in estimates of \( S_{w,ro} \) and \( k_{w,ro} \) for each measured time interval. The sequence of \( S_{w,ro} \) and \( k_{w,ro} \) data pairs can then be used to fit various permeability models. Although Christiansen (2001) warned that data differentiation techniques can cause magnification of errors in the data, the directness of this approach reveals the effects of underlying assumptions and processes on the \( k_{w,t} \) (\( S_w \)) relationship more clearly than numerical procedures that simulate production.

The method introduced by Hagoort (1980) requires three main assumptions: (i) the effect of capillary pressure on the saturation profile is negligible; (ii) the centrifugal acceleration along the length of the sample is constant; and (iii) the gradient in total potential of the nonwetting fluid is negligible. Hagoort (1980) evaluated the impact of the first assumption using the capillary number, \( N_{cg} \), defined as

\[ N_{cg} = \frac{\sigma \sqrt{\phi / k}}{\Delta \rho g L} \]  

The \( N_{cg} \) is an indicator of the relative importance of capillary pressure compared with acceleration due to Earth’s gravitation (or the buoyancy effect). For a centrifugal field, \( g \) in Eq. [19] is replaced by \( a_{avg} \). Saeedi and Pooladi-Darvish (2007) employed a forward-backward numerical modeling loop to show that capillary pressure effects become negligible when \( N_{cg} \leq 10^{-2} \). Above this threshold, the calculated relative permeabilities will be underestimated. Hagoort (1980) has proposed a correction procedure for centrifuge experiments which fail to meet this criterion. Hagoort (1980) also evaluated the effect of the assumption of a constant centrifugal acceleration throughout the sample. Production curves generated for a synthetic porous medium in a rotor-core length configuration of \( r_m/L \geq 2.5 \) to 3 (where \( r_m \) is the radial distance between the center of rotation and the center of the core), did not show any significant deviations from a situation in which the acceleration can safely be assumed constant (e.g., gravitational acceleration at the surface of the Earth).

Numerical Models for Transient, Unsaturated Flow in a Centrifugal Field

Numerical techniques allow flexible modeling of boundary and initial conditions, representation of heterogeneity within the sample, simulation of a sequence of angular velocities, and interpretation of the relative permeability and capillary pressure functions using inverse optimization techniques. All of these features are now available for simulating transient centrifuge experiments and here an overview is presented of parallel developments in the petroleum and soil science communities.

Numerical tools for interpreting multifluid flow experiments in a centrifugal field were first introduced by O’Meara and Lease (1983), O’Meara and Crump (1985), and Hirasaki et al. (1992, 1995). A clear overview of the steps involved in simulating multiphase fluid flow in a centrifugal field using numerical techniques was given by Christiansen (2001). The numerical solutions of multiphase fluid flow equations are frequently called IMPES solutions, which is short for IMplicit Pressure and Explicit Saturation solution (Firoozabadi and Aziz, 1991). Modifications to how the equations are discretized result in more stable and faster semi-implicit (Hirasaki et al., 1992) or fully implicit numerical solutions (Skauge and Poulsen, 2000; App and Mohanty, 2002). Inverse parameter estimation procedures have also been included in various applications of the numerical simulation models (e.g., Skauge and Poulsen, 2000; Chardaire-Riviere et al., 1992).

In soil science, the software code HYDRUS-1D (Šimůnek et al., 1998) is a well-documented and much-used tool for numerically simulating flow and transport in the vadose zone under normal gravity conditions. Šimůnek and Nimmo (2005) adapted the HYDRUS-1D code to simulate transient, unsaturated flow and transport in a centrifugal field. According to Šimůnek and Nimmo (2005), steep pressure and saturation gradients require significant spatial and temporal refinements, particularly in the early part of the centrifuge experiment and at the outflow end face of the sample. To obtain a forward numerical solution, the capillary pressure and relative permeability functions must be specified as part of the model input. The way boundary conditions are defined in HYDRUS-1D is presented below. Variation of the angular velocity during the experiment is accommodated in the software. This allows the simulation of the acceleration of
the centrifuge rotor during the initial startup of the centrifuge experiment and the step-like increase of angular velocity in multi-rate, transient-flow centrifuge experiments. The HYDRUS-1D software includes a Marquardt–Levenberg type parameter optimization algorithm for inverse estimation of soil hydraulic (or solute transport) parameters from measured transient or steady-state flow (or transport) data. The objective function can be defined by the average saturation within a specified volume of the sample, the average pressure, and the outflow of wetting fluid from the core (Šimůnek and Nimmo, 2005).

Factors Influencing Interpretation of Centrifuge Experiments

Compaction

Compaction of unconsolidated porous media is a known side effect of the determination of capillary pressure and relative permeability with centrifuge methods (Omoregie, 1988; Khanzode et al., 2002; Nimmo and Akstin, 1988). The effects of compaction on the capillary-pressure saturation function as a result of increased overburden pressure are well known (Dabbous et al., 1976; Stange and Horn, 2005; Assouline, 2006). The pore size distribution changes as the proportion of large pores is reduced and the number of smaller pores increases. The shift in pore size distribution generally results in an increase in the air-entry pressure, a decrease in the saturated water content, and an increase in the residual water content.

The impact of compaction on the relative permeability curve is less well documented (Blake et al., 1976; Reicosky et al., 1981; Klein et al., 1983). While compaction is known to decrease the saturated hydraulic conductivity, it is not clear how the relative permeability curve determined using centrifugation and expressed as a ratio of $k$ for the compacted material differs from the relative permeability curve determined in Earth’s gravitational field and expressed as a ratio of $k$ for the “undisturbed” material. Because of unsaturated flow conditions, reductions in the number of large pores due to compaction may not show up in the relative permeability curve; however, changes in the shape of the relative permeability curve may occur at low saturations due to increased numbers of small pores.

Nimmo and Akstin (1988) studied the combined effects of mechanical compaction followed by compaction resulting from centrifugation on the unsaturated hydraulic conductivity of Oakley sand samples. By systematically changing variables in the mechanical compaction (i.e., drop height and frequency of impacts) and varying between two angular velocities (84 and 314 rad s$^{-1}$) in the compaction step using the centrifuge, they found that additional compaction as a result of different degrees of centrifugation did not influence the unsaturated hydraulic conductivity as much as the initial mechanical compaction did. Changes in the mechanical compaction resulted in an order of magnitude difference in unsaturated hydraulic conductivity. Compaction as a result of different degrees of centrifugation generally resulted in much less than an order of magnitude difference in unsaturated hydraulic conductivity. The effects of mechanical compaction alone were not reported in their work. An indication that compaction as a result of centrifugation is significant, however, is presented in their Table 2 (p. 305), which documents a considerable reduction in porosity between noncentrifuged samples and those subjected to centrifugation at $\omega = 84$ rad s$^{-1}$. Although the impact of initial compaction by centrifugation is not supported by direct measurements, Nimmo and Akstin (1988) showed through modeling that the effect of initial compaction by centrifugation on unsaturated hydraulic conductivity is smaller than the effect of increasing levels of centrifugation.

To prevent compaction during a centrifuge experiment, methods have been developed to preconsolidate the porous material by using centrifugal force (Nimmo et al., 1987) or loading the porous medium in a pressure cell (Omoregie, 1988; Hirasaki et al., 1992). It could be argued, therefore, that centrifuge measurements are most suitable for studying the hydraulic properties of compacted material from thick vadose zones. Because the extent of compaction is dependent on the angular velocity, the use of “undisturbed” samples in multiple-rate centrifuge experiments might be a way to incrementally determine the relative permeability curve before the entire pore size distribution is impacted, thereby minimizing the effects of compaction.

Effect of Increased Acceleration on Menisci and Capillary Pressure

The characteristics of the interface between two liquids under zero- and normal-gravity conditions have been studied for a wide variety of engineering applications (e.g., Sáez and Carbonell, 1990; Willet et al., 2000; Adams et al., 2002). It is clear from this research that, in the presence of gravity, the shape of liquid bridges deforms with respect to conditions of zero gravity. Assuming that the interfacial properties between fluids and solids remain unchanged, increasing acceleration in these formulations may allow the calculation of the shape of liquid bridges in a centrifugal field. Work by Knight and Mitchell (1996), König et al. (1998), and Schubert (1982) has shown, however, that the contact angle also changes with centrifugal acceleration. In addition to this phenomenon, the dynamic properties of the contact angle play an important role in drainage or imbibition of a wetting fluid. Because the contact angle depends on drainage or imbibition and on the flow rate of fluids (van Brakel and Heertjes, 1977), nonlinearity in the meniscus shape, and consequently in capillary pressure values, is to be expected during centrifuge experiments.

Carrying out capillary rise experiments at different centrifugal accelerations, Depontitis et al. (2001) and Rezzoug et al. (2004) showed that a significant deviation from normal scaling relationships occurs for accelerations greater than $50 \times g$. As possible explanations, they suggested changes in the shape of menisci and capillary pressures with increasing acceleration, as well as resolution issues in the experiments. According to Schubert (1982), changes in the shape of water bridges by gravity can be neglected if the following condition is met:

$$\frac{\Delta \rho \omega d}{P_c} < 10^{-2}$$

where $d$ is a geometrical parameter representing the characteristic length of the water bridges, which is assumed to be smaller than the particle size. According to König et al. (1998), this criterion can also be applied to conditions of increased accelerations. For an air–water system under $1 \times g$ conditions, the water bridge can be expected to be undeformed when particles $< 380 \mu m$ are used.
The diameter of particles in a centrifuge experiment would then have to be smaller than $380/\sqrt{N} \mu m$ at accelerations of $N \times g$ (Petersen and Cooke, 1994). In many cases this condition cannot be met, suggesting that centrifugation deforms the shape of fluid bridges and influences capillary pressure. Additional experimental and theoretical research is needed to evaluate the magnitude of this effect on centrifuge measurements of relative permeability.

Boundary Conditions for Transient, Unsaturated Flow in a Centrifugal Field

Boundary conditions for the analytical Hagoort (1980) method and a number of numerical models are presented in Table 1. The numerical models of Christiansen (2001), Hirasaki et al. (1992), and O’Meara and Crump (1985) all assume zero capillary pressure at the outlet end face. Firoozabadi and Aziz (1991) used different boundary conditions in their IMPES numerical model. The pressure of the wetting fluid was only defined at the inflow end face and the fluxes were equated. Their simulated saturation profiles show that the porous medium near the outlet end face is saturated with the wetting fluid even after long centrifuge times. Thus, the alternative numerical simulation scheme of Firoozabadi and Aziz (1991) may support the assumed zero-capillary-pressure boundary condition at the outlet end face used by other researchers. The HYDRUS-1D code offers various definitions of boundary conditions at the inlet and outlet end faces of the sample depending on the type of centrifuge experiment (single or multiple rate) and if ramp-up of the rotor needs to be simulated.

Wunderlich (1985) performed centrifuge experiments on Berea sandstone samples that were fully saturated with an epoxy (wetting, dense, and low viscosity) that was displaced by air. After spinning the samples for 24 h at different angular velocities (52.4, 104.7, 157.1, 261.8, 575.9, and 837.7 rad s$^{-1}$), the temperature was raised and the epoxy cured in situ. Thin sections, cut parallel and perpendicular to the long axis of the sample, were point counted to determine the spatial distribution of the abundance of the displaced epoxy. Analysis of the experiments showed that the cores at the outlet end were fully saturated with the displaced epoxy at 52.4, 104.7, and 157.1 rad s$^{-1}$. The thickness of this saturated layer decreased with increasing angular velocity. These observations were corroborated later through magnetic resonance imaging (Chen and Balcom, 2005) and nuclear magnetic resonance (Baldwin and Yamanashi, 1991) measurements on centrifuged core samples. Epoxy saturations at the outlet end face of the higher angular velocity experiments were <1, indicating that gas broke through the outlet end face. Through thermodynamic reasoning, Wunderlich (1985) proposed that the surface of zero capillary pressure should be located at the wetting–nonwetting fluid interface in the receiving cup instead of at the outlet end face of the sample. Calculations with an alternative capillary pressure model based on this reasoning showed that saturation at the outlet end face is <1.

O’Meara et al. (1992) highlighted the formation of droplets at the outlet end face of the core. If droplets form, the capillary pressure at the end face is not zero because of drop curvature and the hydrostatic pressure across the drop. O’Meara et al. (1992) investigated the impact of pendant drops on the capillary pressure–saturation curve by estimating the error in average saturations that results from different boundary condition locations. According to their analysis, the effect of pendant drops on the plane of zero capillary pressure is insignificant.

Another issue is the stability of the wetting–nonwetting fluid interface at the outlet end of the core in a centrifugal field. The radius of a capillary tube that would just retain a wetting liquid against centrifugal force would correspond to the radius of the largest pores at the outlet end face of the core, which would still be saturated at this critical centrifugal acceleration. Calculations by Wunderlich (1985) failed to show a relationship between this critical radius and the epoxy displacement observations. O’Meara et al. (1992) considered the stability of a 1- to 1.5-grain diameter thick zone at the outlet end face of the core. They proposed a Bond number ($N_B$) criterion to describe the angular velocity range across which 100% saturation exists. The Bond number, in this particular case, was defined by using a pore radius instead of intrinsic permeability, i.e.,

$$N_B = \frac{\Delta \rho a c, avg r_p^2}{\sigma} \tag{21}$$

with $r_p$ being the radius of the largest pores drained as the entry pressure is exceeded. O’Meara et al. (1992) explored several stability models, the differences among the models being the way in which the geometry of the porous medium and the water

### Table 1. Overview of boundary conditions used in different transient-flow models for the displacement of a viscous, wetting fluid by a non-viscous, nonwetting fluid. Variables are defined in the Appendix.

<table>
<thead>
<tr>
<th>Properties at inlet and outlet</th>
<th>Analytical model†</th>
<th>Numerical model I (IMPES)‡</th>
<th>Numerical model II (IMPES)§</th>
<th>Numerical model III (HYDRUS-1D)¶</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{w,i}$ $P_{nw,i}$</td>
<td>$P_{nw,i} = P_{nw,o} - 0.5P_{nw}\omega^2(r_o^2 - r_i^2)$</td>
<td>$P_{nw,i} = \text{initial } P_{nw}$</td>
<td>$P_{nw,i} = \text{constant } P_{nw}$</td>
<td>$u_{w,i} = -u_{nw,o}$</td>
</tr>
<tr>
<td>$P_{c,i}$ $P_{c,o}$</td>
<td>$P_{c,i} = 0.5\Delta \rho \omega^2(r_o^2 - r_i^2)$</td>
<td>$P_{c,i} = 0$</td>
<td>$P_{c,i} = \text{constant } P_{c,i}$</td>
<td>$u_{w,i} = u_{nw,i}$</td>
</tr>
<tr>
<td>$S_{w,i}$ $S_{nw,i}$</td>
<td>$S_{w,i} = S_{w,t}$</td>
<td>$S_{w,i} = S_{w,o}$</td>
<td>$S_{w,o} = 1$</td>
<td>$u_{w,i} = 0$</td>
</tr>
<tr>
<td>$k_{w,i}$ $k_{nw,i}$</td>
<td>$k_{nw,i} = 0$</td>
<td>$k_{nw,i} = \text{initial } k_{nw,i}$</td>
<td>$k_{nw,i} = \text{constant } k_{nw,i}$</td>
<td>$u_{w,i} = 0$</td>
</tr>
<tr>
<td>$u_{w,i}$ $u_{nw,i}$</td>
<td>$u_{w,i} = 0$, $u_{nw,i} \neq 0$</td>
<td>$u_{w,i} = 0$, $u_{nw,i} \neq 0$</td>
<td>$u_{w,i} = 0$, $u_{nw,i} \neq 0$</td>
<td>$u_{w,i} = 0$, $u_{nw,i} \neq 0$</td>
</tr>
</tbody>
</table>

† Hagoort (1980).
‡ Christiansen (2001), Hirasaki et al. (1992) and O’Meara and Crump (1985).
¶ Šimůnek and Nimmo (2005).
layer is represented (Fig. 5). Calculations with each of the models showed that the critical Bond number below which the water layer is stable varies between 0.5 and 0.84. O’Meara et al. (1992) argued that running experiments under \( N_B \) conditions \( > 0.5 \) results in desaturation of the end face. Under these conditions, alternative definitions of boundary conditions should be tested.

Why these models do not correspond well with observations may be due to inertial forces and vibrations caused by the centrifuge. Equally likely is the possibility that the zero capillary pressure surface is located between the wetting–nonwetting interface and the outflow end face of the core. This will depend largely on the material out of which the end piece is constructed. O’Meara et al. (1992) tested the effect of a 1-mm-thick perforated rubber and Teflon sheet. Teflon, which is nonwetting with respect to water, prevents the formation of water films that could draw the zero capillary pressure plane away from the outlet end face. Rubber can be water wet, which may result in higher production through film flow. The use of a Teflon end piece in air–water displacement experiments is therefore recommended.

**Corrections for the Assumption of One-Dimensional Flow**

All models that are available in the literature for simulating multifluid flow through porous media in a centrifugal field assume one-dimensional flow. Discussions on whether or not this is a valid assumption and on the possible impact of simplifying the three-dimensional flow field are currently not available. The literature dealing with the determination of the capillary pressure function from centrifuge experiments does, however, include a number of publications on the effects of simplifying the three-dimensional equipotential domain to a one-dimensional system. Important insights and conclusions from that work can potentially be equally valid for the determination of the relative permeability function.

Gravity and radial effects have been dealt with rigorously in the studies by Chen and Ruth (1995), Christiansen and Cerise (1992), Christiansen (2001), Forbes et al. (1994), and Forbes (1997) focused on correcting the capillary pressure–saturation functions for radial effects. In an effort to evaluate the effect on relative permeability, we calculated the capillary pressure–saturation functions using the one- and two-dimensional centrifuge models of Christiansen (2001) and converted them to relative permeability functions using the Brooks and Corey (1964) capillary pressure and relative permeability models.

The Microsoft Excel spreadsheet program PcCentSim of Christiansen (2001) was used to construct capillary pressure and
(average) fluid saturation curves. The capillary pressure model of Bentsen and Anli (1977) was used to convert the calculated capillary pressure to average saturation using a threshold pressure $P_t = 6.89 \times 10^3$ kg m$^{-1}$ s$^{-2}$ ($= 70.3$ cm H$_2$O or 1 psi), an irreducible saturation of the displaced fluid $S_{wr} = 0.20$, and a pressure span $\gamma$, which is a characteristic of the pore size distribution of the porous medium. Calculations were performed with $\gamma$ ranging at regular intervals from a porous medium with a uniform pore size ($\gamma = 3.44 \times 10^3$ kg m$^{-1}$ s$^{-2} = 35.2$ cm H$_2$O = 0.5 psi) to a porous medium with a broader pore size distribution ($\gamma = 1.033 \times 10^4$ kg m$^{-1}$ s$^{-2} = 105.5$ cm H$_2$O = 1.5 psi). These parameter values are similar to those used by Christiansen and Cerise (1992). The dimensions and properties of the cylindrical sample were: length, 39.0 mm; diameter, 33.3 mm; and porosity, 0.30. The densities of the sample fluids were 0.997 g cm$^{-1}$ for water and $1.18 \times 10^{-3}$ g cm$^{-1}$ for gas. In the centrifuge rotor, the radial distances to the sample inlet and outlet were 47.0 and 86.0 mm, respectively.

The results of the capillary pressure calculations for $\gamma = 3.44 \times 10^3$ and $1.033 \times 10^4$ kg m$^{-1}$ s$^{-2}$ are presented in Fig. 7a and 7b, respectively. The parameters $P_b$ (bubbling pressure) and $\lambda$ (pore-size distribution index) of the Brooks and Corey capillary pressure model were optimized to fit the constructed Bentsen and Anli capillary pressure saturation function using a nonlinear least squares optimization procedure in the statistical software package SAS (SAS Institute, Cary, NC). The resulting Brooks and Corey capillary pressure curves are also included in Fig. 7a and 7b and show that the correspondence between the two models was acceptable. Relative permeability functions were constructed with the Brooks and Corey relative permeability model using the optimized parameters and assuming $l = 2.0$ and $e = (2 + 3x)/\lambda$ (Fig. 8). It is clear from Fig. 8 that, especially in the lower saturation range, the difference in relative permeability may be up to an order to magnitude. Although this analysis involves a number of steps that introduce errors into the relative permeability function, the observation that relative permeability differs between one- and two-dimensional flow domains is new and warrants further research.

**Corrections for Early-Time Centrifuge Data**

Depending on the complexity of the interpretation model, three corrections are generally applied to the early part of transient outflow data. First, the data should be corrected for the time taken by the centrifuge to build up capillary pressure to the air-entry pressure. Second, the period after air-entry pressure has been reached, and in which the shock front migrates through the sample, needs to be accounted for. Third, the acceleration of the centrifuge rotor during the initial stage of the centrifuge experiment, the so-called ramp-up, changes the production data significantly from that where the centrifuge rotor would instantly be at the required speed. Interpretation techniques that account for capillary pressure, shock front production, and centrifuge ramp-up, but require minimal correction to the raw data, are preferred.

**Time Delay Due to Air-Entry Pressure**

No production of the wetting fluid from the core will take place until the air-entry pressure has been reached. This occurs when the angular velocity exceeds a critical value. For interpretation models, which do not incorporate capillary pressure, the total time from the start of the centrifuge in which no production takes place should be subtracted from all times for which production is registered. Multi-rate centrifuge runs for determining the capillary pressure saturation function can be used to determine the air-entry pressure (Hassler and Brunner, 1945; Firoozabadi et al., 1988; Bentsen and Anli, 1977). The air-entry pressure can then be converted to a corresponding angular velocity and the correction time can be calculated.

**Shock Front Production**

After the air-entry pressure at the inlet has been reached, a drying front (or shock front) moves through the sample. In many analytical and numerical models this shock front is a sharp step in saturation, but in reality, because of the size distribution of the pores, the front is diffuse. The time since starting the centrifuge at which the shock front reaches the outlet end face of the core needs to be subtracted from all following data if the interpretation method does not take into account the production during migration of the shock front. If the intrinsic permeability is measured using an independent laboratory technique, the time taken for
the migration of the shock front through the sample can be calculated (Hagoort, 1980).

Correcting for Ramp-up of the Centrifuge Rotor

Some interpretation models (e.g., Hagoort, 1980) assume that the centrifuge runs at constant angular velocity instantly after starting the experiment. In reality, the rotor accelerates to the required constant angular velocity during a finite period of time. Depending on the type of centrifuge and the final speed required, the acceleration of the rotor may take tens of seconds up to several minutes. Especially for very permeable material, a significant portion of the production will take place during ramp-up. Hirasaki et al. (1995) presented a way to correct for the effect of acceleration, as did Ding et al. (1999), who in addition showed a significant effect on the relative permeability function between corrected and uncorrected experimental data. The strategy followed in both studies was to adjust the time axis for a centrifuge run with ramp-up to dimensionless time. The dimensionless time, \( t_D \), is calculated with the following equation:

\[
{t_D} = \frac{\Delta \rho 0.5 (r_e + r_i)k}{\mu_e \Phi L} \int_0^t \omega^2 (t) \, dt
\]

where \( t \) is the time since the start of the centrifuge experiment. The term in front of the integral is a constant, while the squared angular velocity is integrated over time. The angular velocity during ramp-up can generally be represented by a straight line of the form \( \omega = \omega_0 + \beta t \), with \( \omega_0 \) the angular acceleration (rad s\(^{-2}\)) of the rotor and \( \beta \) the offset (rad s\(^{-1}\)). The integration of angular velocity from \( t = 0 \) until any time \( 0 < t < t_c \) (where \( t_c \) is the end of the ramp-up period) then follows as

\[
\int_0^t \omega^2 (t) \, dt = \frac{\alpha^2}{3} t^3 + \frac{\alpha \beta}{2} t^2 + \frac{\beta^2}{2} t
\]

For the period after ramp-up, \( t > t_c \), the integral of the squared angular velocity up to time \( t \) is the sum of Eq. [23] evaluated at \( t = t_c \) plus the constant angular velocity squared since \( t_c \), i.e.,

\[
\int_0^t \omega^2 (t) \, dt = \frac{\alpha^2}{3} t_c^3 + \frac{\alpha \beta}{2} t_c^2 + \frac{\beta^2}{2} t_c + \omega^2 (t - t_c)
\]

The dimensionless time series of production is analyzed with the method of Hagoort (1980) in the same way as the normal time series of production; however, the calculation of relative permeability is now to be performed with the expression

\[
k_{rw,ro} = \frac{\mu_w \Phi L}{\Delta \rho 0.5 (r_e + r_i)k} \frac{dQ_w}{dt_D}
\]

Experiment Type, Selection of Measurable Variables, and Uniqueness

Multi-rate centrifuge experiments generally decrease uncertainty in the estimated hydraulic functions compared with single-rate experiments. Using an inverse estimation procedure in a numerical fluid flow simulation code, O’Meara and Crump (1985) were able to show that the parameters of the capillary pressure and relative permeability models were estimated with a higher degree of certainty using multi-rate centrifuge experiments than if they had been estimated using a conventional interpretation of the two separate experiments (multi-rate for capillary pressure and single-rate for relative permeability). In contrast, Skauge and Poulsen (2000) found that the capillary pressure and relative permeability functions obtained from multi-rate centrifuge experiments over- and underestimate, respectively, those obtained from long-core gas gravity drainage experiments. The results of single-rate centrifuge experiments with low angular velocity compared better with those of the independent laboratory measurements.

In both single-rate and multi-rate centrifuge experiments, the change from a particular angular velocity to the next is reflected in all time series of measurable state variables (i.e., fluid pressures, saturations, and fluxes). The main calibration data set for determining relative permeability has traditionally been the cumulative volume of fluid produced from the sample. Inclusion in the calibration data set of local saturation or pressure measurements at several locations along the length of the sample has been shown to decrease the uncertainty in the estimated relative permeability function (O’Meara and Crump, 1985; Firoozabadi and Aziz, 1991; App and Mohanty, 2002; Al-Omair and Christiansen, 2005; Chardaire-Riviere et al., 1992). Analyzing the cumulative fluid volume produced during single-rate centrifuge experiments with a numerical simulation code, Firoozabadi and Aziz (1991) found that two completely different relative permeability functions could satisfactorily explain the calibration data. According to these researchers, the inverse simulation of a single-rate centrifuge experiment, therefore, does not lead to a unique relative permeability function.

An important synthesizing modeling effort to establish the importance of local saturation measurements for the determination of the relative permeability function was presented by App and Mohanty (2002). They generated synthetic data consisting of local wetting fluid saturation through time at fixed intervals along the length axis of samples and wetting fluid production through time by using a numerical simulation code. The simulations were performed for three different sets of relative permeability curves at low and high mobility ratios for gas–oil, oil–water, and gas–condensate displacements. Each combination was simulated with

Fig. 8. Relative permeability functions calculated with the Brooks and Corey (1964) relative permeability function using the optimized parameters of the Brooks and Corey (1964) capillary pressure model; \( \gamma \) is the capillary pressure span.
an angular velocity of 52.4 and 314.2 rad s\(^{-1}\). The mobility ratio, \(M\), is defined as
\[
M = \frac{k_{\text{raw}} S_i \mu_w}{k_{\text{rw}} S_i \mu_w}
\]
where \(k_{\text{raw}}\) is the relative permeability of the nonwetting fluid at residual saturation of the wetting fluid and \(k_{\text{rw}}\) is the relative permeability of the wetting fluid at initial saturation of the wetting fluid. The relative permeability curves were subsequently obtained by history matching the synthetically generated data. For each combination, two history matches were performed: one with and one without local saturation data. By comparing the estimated relative permeability curves with those used for generating the synthetic data, the effect of the additional data was evaluated. Their analysis showed that the inclusion of local saturation information improves the relative permeability estimation for oil–water and gas–condensate systems with low mobility ratios (\(M \leq 5\)). For gas–oil systems (\(M = 1000\)) the inclusion of local saturation information did not significantly improve the estimation of the relative permeability of the wetting fluid. App and Mohanty (2002) concluded that gas offers relatively small resistance to flow compared with the oil, so that only the oil relative permeability is required to describe the displacement of oil. Although the mobility ratio for gas–water systems is generally <1000, App and Mohanty (2002) surmised that a similar conclusion can be drawn for such systems.

Al-Omair and Christiansen (2005) performed gas–water displacement experiments using a spinning disk centrifuge. The evolution of the water saturation profile was deduced from video imaging the spinning disk through time. One of their findings was a dependence of the relative permeability function on angular velocity when their numerical simulations were only calibrated to the characteristics of the ramp-up period, \(\alpha = 9.02\) rad s\(^{-2}\) and \(\beta = 0\) (see Eq. [23] and [24]). The simulated ramp-up period lasted for 81.3 s and was represented in the model by increasing the angular velocity stepwise at 2.5-s intervals. In both cases, the simulated centrifuge time was 24 h. It was assumed that the sample was fully saturated with water at the start of the experiment.

The synthetic production data including ramp-up of the rotor were corrected with the method proposed by Ding et al. (1999). The specified properties of the sample and centrifuge rotor were used to calculate the constant term in Eq. [22]. Since the characteristics of the ramp-up period, \(t_r\), \(\alpha\), \(\beta\), and \(\omega\), are all defined, dimensionless time, \(t_{\text{DP}}\), can be calculated, resulting in

![Fig. 9. Synthetic production data generated with forward simulations in HYDRUS-1D modified for centrifugal force fields, simulating a 24-h single-rate transient centrifuge experiment with and without ramp-up of the centrifuge rotor.](image)

Example Application of Transient, Unsaturated Flow in a Centrifugal Field

Here we demonstrate the interpretation of single-rate, transient-flow centrifuge experiments with the analytical model of Hagoort (1980) as modified by Christiansen (2001) and numerical simulations using the modified version of HYDRUS-1D (Šimůnek and Nimmo, 2005).

Two forward numerical simulations were run to generate synthetic production data, one with and one without ramp-up of the centrifuge rotor (Fig. 9). The simulation run that did not incorporate rotor ramp-up was used to verify the performance of the technique proposed by Ding et al. (1999) to correct for the effect of ramp-up. The sample dimensions and hydraulic properties and parameters of the van Genuchten (1980) capillary pressure and relative permeability functions for the material used in the numerical experiments are presented in Table 2. The hydraulic properties and van Genuchten parameters are default values for a silt-textured soil as defined in the soil catalog of HYDRUS-1D. The dimensions of the sample and rotor are characteristic for Beckman ultracentrifuges. The domain was discretized with 1000 cells gradually decreasing in size from a maximum at the inlet end face to a minimum at the outflow end face of the sample. The boundary condition at the inlet end face was a zero constant flux density of the wetting fluid, while at the outlet end face a constant gauge pressure of zero (indicating atmospheric conditions) was defined. The simulated angular velocity was \(\omega = 733.03\) rad s\(^{-1}\), and in the case of ramp-up the acceleration of the rotor \(\alpha = 9.02\) rad s\(^{-2}\) and \(\beta = 0\) (see Eq. [23] and [24]). The simulated ramp-up period lasted for 81.3 s and was represented in the model by increasing the angular velocity stepwise at 2.5-s intervals. In both cases, the simulated centrifuge time was 24 h. It was assumed that the sample was fully saturated with water at the start of the experiment.

### Table 2. Sample (silt soil) and centrifuge properties used in the example calculations.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbols and values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample length</td>
<td>(L = 40.0) mm</td>
</tr>
<tr>
<td>Centrifuge rotor dimensions†</td>
<td>(r_i = 46.0) mm, (r_o = 86.0) mm</td>
</tr>
<tr>
<td>Hydraulic properties‡</td>
<td>(k = 6.32 \times 10^{-14}) m(^2), (\phi = 0.46)</td>
</tr>
<tr>
<td>van Genuchten parameters§</td>
<td>(S_{w*} = 0.074), (\alpha_{w*} = 1.6 \times 10^{-3}) mm(^{-1}), (n = 1.37), (I = 0.5)</td>
</tr>
</tbody>
</table>

† \(r_i\) and \(r_o\), radial distances to the inlet and outlet end faces of the core, respectively. ‡ \(k\), intrinsic permeability; \(\phi\), total porosity. § \(S_{w*}\), residual saturation of the wetting fluid; \(\alpha_{w*}\) and \(n\), parameter and exponent, respectively, of the van Genuchten model; \(I\), pore connectivity parameter.
a corrected production curve (Fig. 10). The shape of this curve closely matches the production vs. time curve obtained with the forward simulation without rotor ramp-up (Fig. 9).

The analytical method of Hagoort (1980) was applied to the two simulated production data sets. Data pairs of fluid saturation and corresponding relative permeability throughout the duration of the experiment were calculated from the derivatives of production with respect to time using the Microsoft Excel spreadsheet program DR_Hagoort of Christiansen (2001) (Fig. 11a). The corrected production curve was analyzed with an adapted version of this program. The corrected data pairs of fluid saturation and corresponding permeability are presented in Fig. 11b. It is clear that the correction proposed by Ding et al. (1999) restores the relative permeability data in the intermediate and high fluid saturation range. A slight discrepancy exists between the estimated relative permeability function and the model-input function (based on the van Genuchten parameter values used in generating the forward simulations) at low fluid saturations. The analytical method underestimates relative permeability in this range because of the assumption of negligible capillary pressure. A forward simulation with $\omega = 1047.19 \text{ rad s}^{-1}$ showed that the relative permeability curve at low saturation was closer to the input curve. This confirms the earlier finding of Skauge and Poulsen (2000) and Christiansen (2001) that the correspondence between estimated and input relative permeability functions improves as $\omega$ increases.

The inverse simulation option of the modified version of HYDRUS-1D was also used to estimate $S_{\text{wr}}, \alpha_{\text{vG}}, n$, and $l$ of the van Genuchten water retention and relative permeability models from the simulated production curve with ramp-up. The porosity and intrinsic permeability were not optimized because, in most practical cases, these parameters are measured independently. The data set used in the objective function of the inversion consisted of the complete simulation output (fluid flux density at the outflow end face through time) of the forward simulation with rotor ramp-up. The structures of the inverse models were exactly the same as those used in the forward simulations. In reality, the true parameter values are unknown and may vary considerably. Therefore, 20 inversion runs were performed to study the impact of initial parameter values on the final parameter estimates. The initial value of each parameter ($S_{\text{wr}}, \alpha_{\text{vG}}, n$, and $l$) was varied individually by $-25, -10, 10, 25$, and $100\%$ of the value used in the forward model, while the remaining parameters maintained their original values. In the subsequent inversions, all parameters were allowed to be optimized.

Results of the 20 inversion experiments are presented in two ways: (i) the ranges in optimized parameter values, sums of squared residuals, and water balance errors at the end of the simulation, as well as the number of successful inversions for each parameter (Table 3), and (ii) the extremes of the relative permeability curves that were obtained from the inversion experiments (Fig. 12). The water balance errors and sums of squared residuals of the simulations were acceptable and consistently low,
although water balance errors show a considerable range. Two inversion runs with initial values of \( n \) close to one (−10 and −25\% of the original value) failed to converge and were therefore considered unsuccessful. The matrix of ranges in optimized values clearly shows that changes in the initial value of one parameter influence the final optimized values of all parameters. This was probably caused by the presence of two or more local minima in the objective function. Additional calibration data might improve the uniqueness of the inverse solution.

Of the 18 relative permeability curves obtained from the numerical inversion experiments, the two most extreme ones encompass a region in which any solution obtained with numerical inversions will probably fall (Fig. 12). The nonuniqueness of these results suggests that inversions based on the production curve using numerical models may overestimate the relative permeability function. Although the extent of this overestimation is probably not well characterized by our limited number of inversion experiments, it is interesting to locate the corrected relative permeability curve obtained from the analytical analysis with the method of Hagoort (1980). The comparison shows that the latter curve plots outside the lower outline of the region in which the numerical inversion results might fall. A tentative conclusion is that the relative permeability curve obtained with the Hagoort (1980) method could be of use as a criterion for successful inversion or could assist in estimating the initial parameter values. The method of Hagoort (1980) might not be an adequate substitute for a numerical inverse simulation combined with a thorough investigation of uniqueness. Its simplicity, ease and speed of application, and, most importantly, model-independent generation of relative permeability functions, however, can be considered as positive arguments in its favor.

### Concluding Remarks

Steady-state and transient-flow centrifuge methods have been used extensively in the soil and petrophysical sciences to determine the relative permeability functions of porous media. In the soil physics community, interest in transient-flow centrifuge experiments appears to be limited, even though advantages such as a relatively short experiment duration, extension of multiphase flow experiments to the low saturation range, and similarities with transient outflow experiments commonly used in soil physics to determine relative permeability may outweigh disadvantages like compaction, menisci deformation, and the less straightforward interpretation. Some important lessons can be learned from the petrophysical literature, which point to a number of outstanding issues for future research, the findings of which could benefit both communities.

Several studies in the petrophysics literature (e.g., Skauge et al., 1997; Skauge and Poulsen, 2000) have indicated that the relative permeability function obtained with transient-flow centrifugation compares well with independent laboratory measurements. A direct comparison between measurements of relative permeability determined using the transient-flow centrifuge method with similar data for the same materials obtained using the IFC or UFA steady-state centrifuge methods would provide valuable insight into the accuracy of the different interpretive approaches used.

A thorough evaluation of the magnitude of error associated with determining the relative permeability using the Hagoort (1980) method might facilitate a broader acceptance of this method. In his original study, Hagoort showed that the analytical approach has a powerful predictive component. Christiansen (2001) adapted the equations of the Hagoort (1980) method in such a way that each interval on the measured production curve is converted to saturation and relative permeability pairs. This is a unique and underexplored aspect of the method, as no assumptions have to be made regarding the underlying relative permeability model.

Presently, there are a large number of numerical simulation software codes available that simulate multiphase flow through porous media in a centrifugal field. With the recent modifications to the HYDRUS-1D code, a versatile software tool became available for interpreting single-rate or multi-rate centrifuge experiments. This software not only allows simulation of varying angular velocity through time but also flexible definition of initial and boundary conditions, the representation of heterogeneity, and solute transport. Besides these options, it offers the inverse simulation of state variables through which optimized model parameters can be found. Benchmarking of software tools that are or could be adapted for centrifugal conditions (e.g., ECLIPSE, HYDRUS-1D, and STOMP) would establish modeling standards and practices that ensure correct simulations.

It is clear from the petrophysical literature (e.g., App and Mohanty, 2002) that for fluid systems with a high mobility ratio (i.e., a large contrast in viscosity and a relatively small contrast in end-point permeabilities), such as in water–gas displacements, the inclusion of local saturation information does not significantly improve the estimation of the relative permeability of the

### Table 3. Range in initial and optimized values (minimum and maximum) of the van Genuchten parameters for the 20 inversion runs. The range of water balance errors, sums of squared residuals of the final optimization run (SSR), and number of successful simulations are also presented.

<table>
<thead>
<tr>
<th>Parameter†</th>
<th>Range in initial values</th>
<th>Optimized ( S_{wr} ) range</th>
<th>Optimized ( \alpha_{VG} ) range</th>
<th>Optimized ( n ) range</th>
<th>Optimized ( f ) range</th>
<th>SSR</th>
<th>Water balance error</th>
<th>Successful simulations‡</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{wr} )</td>
<td>0.055 – 0.147</td>
<td>0.068 – 0.079</td>
<td>1.45 \times 10^{-3} – 1.69 \times 10^{-3}</td>
<td>1.37 – 1.37</td>
<td>0.47 – 0.57</td>
<td>1.85 \times 10^{-3} – 2.17 \times 10^{-3}</td>
<td>0.02 – 0.16</td>
<td>5</td>
</tr>
<tr>
<td>( \alpha_{VG} )</td>
<td>1.20 \times 10^{-3} – 3.2 \times 10^{-3}</td>
<td>0.069 – 0.096</td>
<td>1.4 \times 10^{-3} – 3.36 \times 10^{-3}</td>
<td>1.36 – 1.37</td>
<td>0.27 – 0.56</td>
<td>1.84 \times 10^{-3} – 2.41 \times 10^{-3}</td>
<td>0.073 – 0.15</td>
<td>5</td>
</tr>
<tr>
<td>( n )</td>
<td>1.027 – 2.74</td>
<td>4.1 \times 10^{-3} – 0.095</td>
<td>2.11 \times 10^{-3} – 0.146</td>
<td>1.36 – 1.37</td>
<td>0.25 – 0.56</td>
<td>1.83 \times 10^{-3} – 1.91 \times 10^{-3}</td>
<td>0.066 – 0.13</td>
<td>3</td>
</tr>
<tr>
<td>( f )</td>
<td>0.375 – 1.0</td>
<td>0.053 – 0.105</td>
<td>1.47 \times 10^{-3} – 3.81 \times 10^{-3}</td>
<td>1.37 – 1.38</td>
<td>0.11 – 0.55</td>
<td>1.84 \times 10^{-3} – 2.02 \times 10^{-3}</td>
<td>0.096 – 0.16</td>
<td>5</td>
</tr>
</tbody>
</table>

† \( S_{wr} \) residual saturation of the wetting fluid; \( \alpha_{VG} \) and \( n \), parameter and exponent, respectively, of the van Genuchten model; \( f \), pore connectivity parameter. ‡ Out of five.
wetting fluid. To support these findings, more water–gas displacement centrifuge experiments with local saturation and outflow measurements on different textured soils should be performed.

The way boundary conditions are defined and the values that they are assigned can significantly influence the simulation results. As shown in experimental work by Wunderlich (1985) and O’Meara et al. (1992), the outflow end face of the core remains saturated with the wetting fluid up to a critical Bond number. Above this critical value, the stability of the interface between the wetting and nonwetting fluid breaks down and the outflow end face desaturates. This seemingly transient behavior of the boundary condition at the outflow end face is not simulated in most numerical models and may negatively influence estimation of the relative permeability function. Errors in the relative permeability function also stem from the fact that all the models assume one-dimensional flow. The effects of simplifying the outflow boundary conditions and neglecting the second and third dimensions require more fundamental research.

Appendix: Definitions

Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>surface area of cross-section perpendicular to the length axis of sample, [L^2] m^2</td>
</tr>
<tr>
<td>a_c</td>
<td>centrifugal acceleration, [L T^-2] m s^-2</td>
</tr>
<tr>
<td>C</td>
<td>constant used in accounting for air pressure, [M L^-1 T^-2] kg m^-1 s^-2</td>
</tr>
<tr>
<td>d</td>
<td>geometrical parameter representing the dimension of the water bridg</td>
</tr>
<tr>
<td>E</td>
<td>es, [L] m</td>
</tr>
<tr>
<td>E_b</td>
<td>total energy of a rotating object, [M L^-1 T^-2] kg m^-1 s^-2</td>
</tr>
<tr>
<td>E_p</td>
<td>pressure potential energy, [M L^-1 T^-2] kg m^-1 s^-2</td>
</tr>
<tr>
<td>g</td>
<td>Earth’s gravitational acceleration, [L T^-2] m s^-2</td>
</tr>
<tr>
<td>I</td>
<td>moment of inertia, [M L^2] kg m^2</td>
</tr>
<tr>
<td>k</td>
<td>intrinsic permeability, [L^2] m^2</td>
</tr>
<tr>
<td>k_p</td>
<td>relative permeability</td>
</tr>
<tr>
<td>k_w</td>
<td>relative permeability of the wetting fluid</td>
</tr>
<tr>
<td>k_ww</td>
<td>relative permeability of the nonwetting fluid</td>
</tr>
<tr>
<td>L</td>
<td>length of sample, [L] m</td>
</tr>
</tbody>
</table>

\[ l \]  pore connectivity parameter
\[ M \]  mobility ratio of two immiscible fluids
\[ N \]  multiplier of Earth’s gravitational acceleration
\[ n \]  exponent in the soil water retention model of van Genuchten
\[ N_g \]  capillary number
\[ P_c \]  fluid pressure, [M L^-1 T^-2] kg m^-1 s^-2
\[ P_b \]  bubbling pressure in Brooks and Corey model, [M L^-1 T^-2] kg m^-1 s^-2
\[ P_t \]  threshold pressure in the Bentsen and Anli model, [M L^-1 T^-2] kg m^-1 s^-2
\[ P_r \]  capillary pressure, [M L^-1 T^-2] kg m^-1 s^-2
\[ Q \]  cumulative production of wetting fluid, [L^3] m^3
\[ q_w \]  fluid flux density, [L T^-1] m s^-1
\[ R \]  radius of curvature, [L] m
\[ r \]  radial distance of object from center of rotation, [L] m
\[ r_p \]  radius of the largest pores drained as the entry pressure is exceeded, [L] m
\[ r_m \]  radial distance between the center of rotation and the center of the core, [L] m
\[ S \]  saturation (fraction of pore space occupied by a fluid)
\[ S_{w} \]  residual or irreducible saturation of the wetting fluid
\[ S_{n} \]  critical saturation of the non-wetting fluid
\[ S_{m} \]  saturated saturation (fraction of pore space occupied by a fluid)
\[ S_{w} \]  saturation of the wetting fluid
\[ S_{w_{c}} \]  saturation of the wetting fluid
\[ S_{n_{c}} \]  saturation of the non-wetting fluid
\[ SSR \]  sum of squared residual between observed and simulated production data, [L^2] cm^2
\[ t \]  time, [T] s
\[ t_c \]  time needed to accelerate centrifuge rotor to required angular velo |
\[ t_d \]  dimensionless time
\[ u \]  fluid volumetric flux, [L^3 T^-1] m^3 s^-1
\[ \alpha \]  angular acceleration of the circular motion, [T^-2] s^-2
\[ \alpha_{eG} \]  parameter in the soil water retention function of van Genuchten,
\[ \beta \]  [L^-1] mm^-1
\[ \beta \]  offset in linear equation describing ramp-up of the centrifuge rotor,
\[ \beta \]  pressure span in the Bentsen and Anli model, [M L^-1 T^-2] kg m^-1 s^-2
\[ \gamma \]  exponent used in the Brooks and Corey relative permeability model
\[ \Theta \]  contact angle, [angle] rad
\[ \chi \]  pore-size distribution index in the Brooks and Corey model
\[ \mu \]  dynamic viscosity, [M L^-1 T^-1] kg m^-1 s^-1
\[ \rho \]  fluid density, [M L^-3] kg m^-3
\[ \sigma \]  interfacial tension between two fluids, [M L^-1 T^-2] kg m^-1 s^-2
\[ \Phi \]  total pressure potential, [M L^-1 T^-2] kg m^-1 s^-2
\[ \Phi \]  total porosity
\[ \omega \]  angular velocity, [angle] rad s^-1

Subscripts

avg average property
o,i outlet or inlet end face of the core
w,nw wetting or non-wetting fluid
sw saturation of the wetting fluid

Superscripts

Si initial saturation
Sr residual saturation

Acknowledgments

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