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Fractal and Multifractal Models Applied to Porous Media

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In the context of this special section of the *Vadose Zone Journal*, fractals can be thought of as binary geometrical patterns that repeat themselves (either exactly or stochastically) over a range of spatial scales. As a result, their properties tend to scale in a power law fashion. Multifractals, in contrast, are geometrical patterns produced by continuously distributed variables whose properties exhibit a multitude of power law scaling relations. The practical value of fractal and multifractal models lies in their ability to effectively simulate the scale-dependent heterogeneity that is typically present in natural systems.

Fractals and multifractals owe most of their current popularity to Mandelbrot (1982). This seminal work established a new geometrical paradigm based on the scaling of heterogeneous systems. The term *fractal* was derived from the Latin word *fractus*, meaning fragmented or irregular, and so it is perhaps not surprising that porous media have played an important role in the development and application of fractal models. In one of the earliest studies, Avnir and coworkers (Avnir et al., 1983; Pfeifer et al., 1983; Pfeifer and Avnir, 1983) used fractal geometry to describe the surface heterogeneity of particulate materials. During the next decade, fractal models were applied to many soil physical properties and processes, including bulk density scaling, brittle fracture, fragmentation, aggregate, particle and pore shapes and size distributions, microtopography, water retention, macropore flow, and solute transport phenomena (Perfect and Kay, 1995).

One of the first applications of multifractal formalism to porous media was by Folorunso et al. (1994). These authors normalized and interpreted micropenetrometer data as a probability measure in an effort to identify scale dependencies in the spatial variability of soil strength. Since then, interest in multifractals has grown to such an extent that multifractal approaches now make up approximately 20% of the papers published on fractal geometry applied to soils (ISI Web of Science, 2008). Altogether, over 1300 scientific papers have been published on the subject of fractal or multifractal methods applied to porous media. This total includes approximately 130 publications that have appeared on this topic in just the past 18 months (ISI Web of Science, 2008).

Given the current high level of interest in the use of fractal geometry to characterize natural porous media, the guest editors decided to organize this special section of the *Vadose Zone Journal* to expose established fractal analysis techniques and cutting-edge new developments to a wider Earth science audience. The core papers were selected from talks and posters presented at PEDOFRACT 2007, an International Workshop on Scale Dependencies in Soil and Hydrologic Systems organized by the Fractal Applications Group from the Technical University of Madrid, Spain. Previous meetings organized by this group resulted in special issues of the journals *Ecological Modelling* (Martín et al., 2005) and *Geoderma* (Pachepsky et al., 2006) focused on fractals applied to soils. PEDOFRACT 2007 was held on 3–6 July 2007 in El Barco de Avila, Spain, and attracted approximately 30 participants from seven different countries. In addition to selected papers from that meeting, contributions were also invited from a few well-known researchers in the field. All of the submitted papers were subjected to a rigorous peer review process. The outcome was this special section, which contains 11 original contributions covering a wide range of fractal and multifractal applications to porous media.

The papers in this special section utilize a variety of research methods, including fractal and multifractal characterization of experimental data, analytical, numerical and lattice Boltzmann modeling, and inverse estimation procedures. The guest editors have grouped the papers into five common themes: theory development, soil fragmentation, pore space geometry, column-scale flow and transport, and field-scale flow and transport.

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Except for the section on theory, which appears first because of its general applicability, the topics are arranged in order of increasing spatial scale.

In terms of theory development, Yu et al. (2009) provide an overview of the physical properties of ideal fractal porous media and explain how natural heterogeneous materials can exhibit both mass- and pore-fractal scaling. Cihan et al. (2009) present new analytical models for predicting the saturated hydraulic conductivity based on the Menger sponge mass fractal. They tested their model predictions against lattice Boltzmann simulations of flow performed in different configurations of the Menger sponge.

Next, Bird et al. (2009) and Martín et al. (2009) introduce some new mathematical approaches for modeling the dynamic fragmentation of earth materials. Bird et al. (2009) present a generalization of Turcotte's (1986) fractal fragmentation model by introducing a time component and relaxing the requirement for a single scale-invariant probability of failure. Martín et al. (2009) show how an extension of Kolmogorov's (1941) fragmentation theory can generate multifractal mass-size distributions. One of the missing links in this special section is the connection between soil fragmentation and pore space geometry, that is, packing models. No papers were submitted on this topic, which definitely deserves more attention if, for example, we want to predict pore-size distributions from fragment-size distributions.

Characterization of pore-space geometry is critical to the development of more physically based models for dynamic soil processes. Paz Ferreiro et al. (2009) apply multifractal formalism to the analysis of nitrogen adsorption isotherm data for samples from different land use management practices implemented on two different soil types. In the past, the soil was often treated as a "black box" with respect to the prediction of flow and transport. With the availability of advanced digital image acquisition and processing capabilities, however, it is now possible to visualize and quantify pore networks. In this context, Kravchenko et al. (2009) perform multifractal analyses on binary images of pores and particles obtained by thresholding two-dimensional X-ray computed tomography images of soil aggregates, while Papadopoulos et al. (2009) merge imaging data collected at different spatial scales to predict a single global pore-size distribution.

At the column-scale, Luo and Lin (2009) used two- and three-dimensional pore fractal and lacunarity analyses to relate solute tracer patterns to soil structural conditions. Tracer distributions and macropore networks were scanned at different locations along large intact soil column using micro-X-ray computed tomography. San José Martínez et al. (2009) used solute breakthrough curve data from previously published column studies to evaluate a numerical form of the fractional advection–dispersion equation (relative to the performance of the standard advection–dispersion equation). Although the fractional advection–dispersion equation is more general than the standard advection–dispersion equation, it does not always provide a better fit. Moreover, theoretical developments are needed to relate the fractional order of differentiation in this model to fractal parameters associated with soil pore space geometry.

The studies by Ozdemirtas et al. (2009) and Cello et al. (2009) both focus on numerical simulations of field-scale flow and transport processes. Ozdemirtas et al. (2009) investigated the effects of fractal surface roughness on mud entry into a single fracture that intercepts a borehole during drilling operations. In contrast, Cello et al. (2009) worked with a discrete fracture network comprised of multiple smooth-walled fractures with a power law distribution of lengths. Constant rate pumping test simulations were then used to estimate noninteger flow dimensions, which the authors were able to relate to the connectivity and fractal geometry of the fracture network.

The use of fractal geometry to characterize and model heterogeneous porous media is an extremely active area of investigation in many different disciplines. By summarizing current research on fractal models applied to porous media, as well as introducing several novel theoretical developments, the 11 papers in this special section should help to further advance this multidisciplinary endeavor. The guest editors sincerely hope that at least some of the *Vadose Zone Journal* readership find these contributions interesting enough to consider using fractal or multifractal techniques to analyze their own complex datasets.

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