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Key Points:

- Fracture clusters cannot be delineated by using average spacing values
- Lacunarity is a parameter that can delineate scale-dependent clustering
- Lacunarity has been used to identify fracture clusters and their organizations

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A technique for revealing scale-dependent patterns in fracture spacing data

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Abstract Data on fracture spacing along scan lines have been widely analyzed for the purposes of characterization. Most of these studies, however, either consider the cumulative frequency of spacing data without regard to the actual sequence of the spacing values or compute an average spacing that may not work for clustered fractures. The coefficient of variation parameter is often used to differentiate between clustered, random, and anticlustered fractures in a scan line but does not address the issue of scale-dependent variations in spacing. Lacunarity is a parameter that has been previously used for delineating scale-dependent clustering in fracture networks with similar fractal dimensions. This technique has the further capability of identifying scales at which different patterns emerge within the same data set. Lacunarity can also delineate possible fractal behavior. This paper tests the ability of lacunarity to find patterns (fractal/uniform/random) within synthetic and natural fracture clusters. A set of four model scan lines (uniformly spaced fractures, periodically spaced fracture clusters, fractal fracture clusters, and random fractures) was considered. The first derivative of the lacunarity curves of these models was used to find the intercluster distance and organization of fractures within the clusters. The same technique was then applied to a set of two natural fracture scan line data, one with fracture clusters with fractal organization within and the other with randomly spaced fractures. It was found that the proposed technique could discriminate between the random and clustered patterns, find the intercluster distance, and identify the type of spatial organization within the clusters.

1. Introduction

Quantifying fracture spacing is the key to understanding the spatial organization of fracture networks and serves as a preliminary step toward stochastic modeling. Previous researchers have employed various parameters for studying fracture spacing that include the coefficient of variation [*Gillespie et al.*, 1999] and fracture spacing index [*Narr and Suppe*, 1991]. However, these descriptors fail to capture the entire range of heterogeneity mainly because they look at data sets only at a single scale. For example, the coefficient of variation determines whether fractures are clustered on the entire length scale of a scan line while the fracture spacing index focuses on the average fracture spacing. Many naturally occurring fractures, however, display heterogeneity such that not only do they occur as clusters but also may have a different organization (e.g., random or fractal) within the clusters. Therefore, while a simple parameter like average spacing is sufficient for predicting the presence of evenly spaced fractures in a wellbore, it will not work where fractures are present in clusters. Also, while a single-scale clustering index may help determine if a fracture set is clustered, it cannot quantify the intercluster distance or find the organization within such clusters.

Semivariograms [*La Pointe and Hudson*, 1985; *Chiles*, 1988] and, more recently, lacunarity [*Roy*, 2013] and the correlation dimension and Lyapunov exponent [*Riley et al.*, 2011], have been introduced as mathematically rigorous parameters that can determine the heterogeneity of fracture data sets at different scales. The focus of the present study is on the quantification of scale-dependent clustering in scan line data using lacunarity. Lacunarity is a parameter developed for multiscale analysis of spatial data and allows for the determination of scale-dependent changes in spatial structure. Stated simply, lacunarity characterizes the distribution of spaces or gaps in a pattern as a function of scale and can thus quantify scale-dependent clustering in a data set. It has been demonstrated by *Plotnick et al.* [1996] that lacunarity versus scale curves of one-dimensional sets will have distinct breaks in slope corresponding to distinct scales within the sets. This technique is therefore well suited for capturing the entire range of heterogeneity in fracture spacing data that may be clustered at one scale while random or even fractal at another.

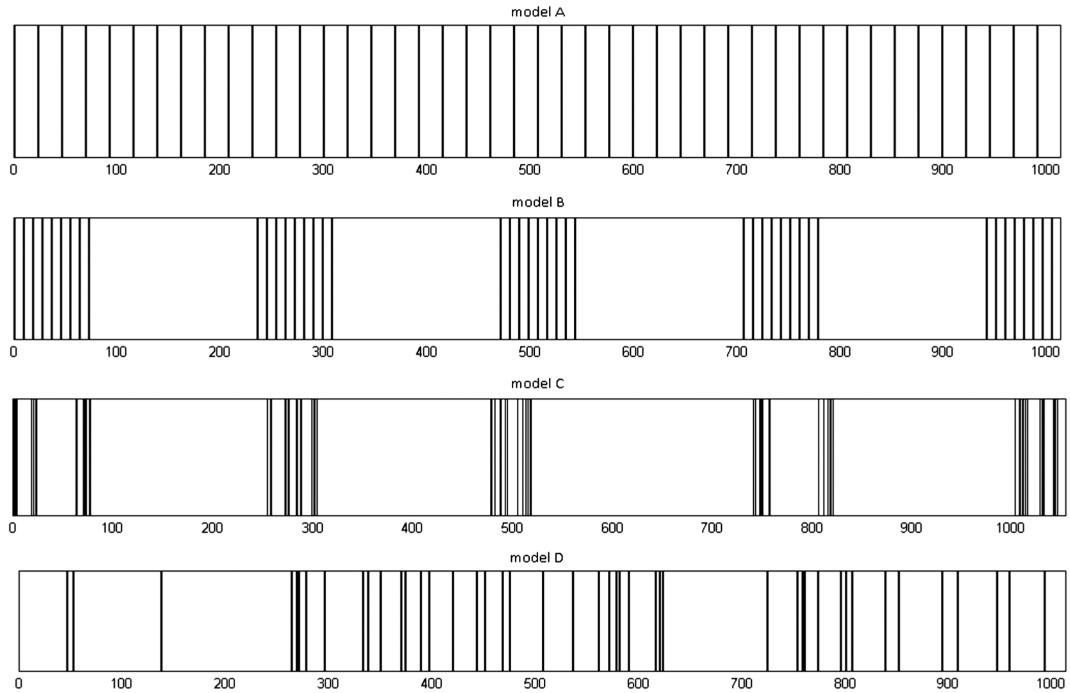

Figure 1. Model scan lines where the *x* axis denotes positions of fractures along a line and the *y* axis denotes presence ($y = 1$) or absence ($y = 0$) of fractures. Model A: uniformly spaced fractures, spacing = 22 units. Model B: equally spaced uniform (spacing = 8 units) clusters spaced at 162 units, cluster width = 73 units. Model C: regularly spaced fractal (Cantor bar, $D = 0.63$) clusters, average spacing = 162 units. Model D: randomly spaced fractures with average spacing = 21 units.

The present research focuses on the application of this technique in revealing changes in scale-dependent patterns in 1-D fracture spacing data. We consider four model scan lines with differences in scale-dependent patterns and generate their lacunarity curves in log-log space to test if the curves can delineate the differences. We further introduce the concept of the first derivative of the log-transformed lacunarity and demonstrate that this function can determine the intercluster spacing and find possible fractal behavior over certain scales. Finally, we test the technique on a set of two natural scan lines, one that comprises fractures occurring in regularly spaced fractal clusters and another that has randomly spaced fractures.

2. Method Development

2.1. Synthetic Scan Lines

Four synthetic scan lines with spacing data (Figure 1 and Table 1) were constructed representing different types of heterogeneities encountered in nature. Model A is a set of fractures spaced equally at 22 length units and representing a homogeneous distribution in space which is typical of "stratabound" fractures found in mechanically layered rock units [*Odling et al*., 1999; *Riley et al*., 2011]. Model B is set of five 73 unit-wide fracture clusters spaced at 162 units with fractures within each cluster spaced at 8 units. The NS trending fractures in the map from Telphyn Point, Wales, [*Rohrbaugh et al*., 2002] subsequently analyzed by *Roy et al*. [2010] display similar regularly spaced clusters with somewhat uniformly spaced fractures within the clusters. Model C is also a set of five 81 unit-wide fracture clusters with intercluster spacings of 172, 142, 182, and 152 units (average intercluster spacing 162 units). The fractures within each cluster, however, are modeled by a randomized Cantor bar, a fractal model with theoretical fractal dimension of 0.63. Cantor bars have been used in modeling fractures by numerous researchers including *Velde et al*. [1990], *Gillespie et al*. [1993], *Barton* [1995], *Chiles* [1988], and *Kruhl* [2013]. Model C was created by integrating properties of this fractal and that of model B and serves as an example of a scan line with "clusters of fractures within clusters" [*Boadu and Long*, 1994]. Finally, model D is a set of fractures whose spacing values were picked at random from a uniform distribution and represents a set of random fractures. Models B, C, and D are typically found in mechanically nonlayered rocks and are examples of nonstratabound fractures [*Gillespie et al*., 1999].

Table 1. Model and Estimated (From Lacunarity Analyses) Fracture Organization Parameters: Fracture Spacing, Intercluster Spacing, Spacing Within Clusters, and Fractal Dimension (*D*)

Scan line	Organization	Parameter (Model)	Parameter (Estimated)
Model A	periodic fractures	fracture spacing = 22	fracture spacing = 22
Model B	periodic clusters (periodic within)	intercluster spacing = 162, spacing within clusters = 8	intercluster spacing = 161, spacing within clusters = 8
Model C	periodic clusters (fractal within)	intercluster spacing = 162, *D* of clusters = 0.63	periodic at scales > 170, *D* of clusters = 0.60
Model D	random fractures	n/a[a]	n/a[a]
Data P11	periodic clusters (fractal within)	n/a[a]	periodic at scales > 0.74 m, *D* of clusters = 0.52
Data P13	random fractures	n/a[a]	n/a[a]

[a]N/a: not applicable.

Gillespie et al. [1999, 2001] studied the spacings of veins and used the coefficient of variation, C_v, for detecting clustering. This parameter is defined as the ratio of the standard deviation to the mean value of the spaces such that $C_v = 0$ for perfectly periodic fractures, $C_v = 1$ for a random distribution, $C_v > 1$ for clustered fractures, and $C_v < 1$ for anticlustered fractures. Models A and D which are periodic and random patterns yielded C_v values of 0 and 1.2, respectively. Models B and C have C_v values of 2.04 and 2.43, respectively, which are consistent with clustered fractures; however, the exact form of the clustering is not revealed by these values.

2.2. Lacunarity and Its First Derivative

A useful conceptual perspective for understanding lacunarity is to evoke the idea of homogeneity. Consider a uniform sequence of alternating zeros and ones like 101010101… and so on. This sequence will map onto itself if a copy is made and moved over by two digits so that the original cannot be distinguished from the translated copy. Another pattern like 1000100010001….and so on will similarly map onto itself if its copy is moved over by four digits. Such homogenous sequences have minimal values in terms of lacunarity because all of the gap sizes (denoted by zeroes in our example) are the same. This behavior is not observed in the case of a slightly more heterogeneous sequence, such as 101000101… where the gaps have a range of sizes including a cluster of three gaps in the middle. Lacunarity quantifies this deviation of a pattern from homogeneity. It is a scale-dependent parameter because sets that are uniform at a coarse scale might be heterogeneous at a finer scale, and vice versa. Lacunarity can thus be considered a scale-dependent measure of textural heterogeneity [*Allain and Cloitre*, 1991; *Plotnick et al*., 1993].

Quantifying lacunarity as a function of scale can be achieved by using the gliding box algorithm [*Allain and Cloitre*, 1991; *Plotnick et al*., 1996]. For a one-dimensional sequence of zeros and ones, this algorithm slides a ruler of a given length, *r*, translated in increments of a unit length such that the total number of steps is given by $(r_t - r + 1)$, r_t being the length of the entire sequence. The number of occupied sites, *s*(*r*), denoted by ones and contained within the interrogator box at each step is calculated, and a distribution of this parameter at the scale *r* is obtained by sliding the ruler through all the steps. Finally, the mean, $\bar{s}(r)$, and variance, $ss^2(r)$, of this distribution are used for calculating the lacunarity, *L*(*r*), at scale, *r* as

$$L(r) = ss^2(\mathrm{r}) / [\bar{s}(r)]^2 + 1 \tag{1}$$

Log-transformed values of lacunarity, log *L*(*r*), plotted against log-transformed values of the scale, log *r*, yield a curve that is characteristic of the heterogeneity of the sequence under investigation. If ϕ is the fracture intensity, defined by number of fractures per unit length of a scan line [*Ortega et al*., 2006], it may easily be proved that $L(1) = 1/\phi$ and $L(r_t) = 1$, such that scan lines with different fracture intensities will have different *L*(1) values. A uniform sequence like 101010…and so on will have $L(r) = 1$ at all *r* values. As pointed out by *Plotnick et al*. [1996], distinct breaks in the slope of this curve correspond to distinct scale-dependent changes within the sequence. Since fractal patterns are scale-independent, they appear to have the same pattern at all scales and produce straight lines in the log *L*(*r*) versus log *r* space.

Plotnick [1995] and *Plotnick et al*. [1996] cited the example of a sequence of randomly spaced clusters and how changes in the slope of the lacunarity curve corresponded to changes in the pattern with scale. A visual inspection of the lacunarity curve, however, is not sufficient for identifying these breaks in scale. In this

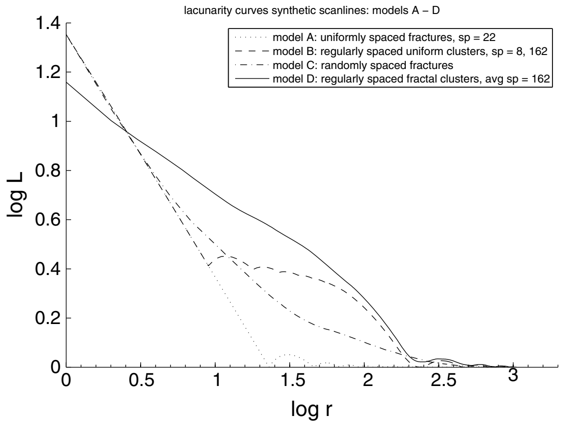

Figure 2. Lacunarity curves for model scan lines A, B, C, and D. Note the range of scales over which model B coincides with model A showing that B is uniform over that range.

paper, we therefore introduce the concept of the first derivative of the lacunarity curve. At each point *i*, the local slope of the log *L*(*r*) versus log *r* curve is found by

$$\mathrm{d}[\log L(r_i)]\,/\mathrm{d}[\log r_i] = \frac{\log L(r_{i+1}) - \log L(r_{i-1})}{\log r_{i+1} - \log r_{i-1}} \tag{2}$$

This value plotted against log *r* yields a curve that is easier to interpret because breaks in the slope of the log *L*(*r*) versus log *r* curve appear as distinct peaks and troughs along a line parallel to the *x* axis.

From the equation established by *Allain and Cloitre* [1991] it can be shown that the lacunarity *L*(*r*) at a scale, *r*, and fractal dimension, *D*, of a one-dimensional fractal sequence are related as

$$L(r) = k(r)^{D-1} \tag{3a}$$

Here *k* is any constant. Taking the logs of both sides, equation (3a) can be transformed to

$$\log L(r) = \log k + (D-1)\log(r) \tag{3b}$$

Finally, differentiating equation (3b) as in equation (2) will yield

$$\frac{\mathrm{d}[\log L(r)]}{\mathrm{d}[\log r]} = D - 1 \tag{3c}$$

For a fractal sequence, therefore, the local slope in equation (3c) when plotted against log *r* will yield a straight line parallel to the *x* axis with a constant value of $D - 1$. A uniform pattern on the other hand, like 101010…and so on, will also plot as a straight line along the *x* axis but with slope equal to zero, such that *D* is actually the embedding Euclidean dimension in this case.

3. Application to Model Scan Lines

Figure 2 shows the lacunarity curves for models A–D. The lacunarity curve of model A drops to zero at log $r \sim 1.35$, i.e., $r \sim 22$ and continues along the *x* axis thereafter. This behavior is indicative of the fact that the fractures in model A are uniformly spaced at 22 units. Model B follows model A up to $r \sim 8$ units and then diverges and follows the lacunarity curve of model C. This happens because at that scale model B behaves as a uniform sequence (like model A) with a constant fracture spacing of eight units while at a larger scale it appears somewhat similar to model C with its uniformly spaced fracture clusters as seen in Figure 1. Model C is linear within the range of the fractal clusters (i.e., between 1 and 81 units). Fitting equation (3b) to the lacunarity data over this range of *r* values using linear regression yielded an estimate of $D = 0.6$ with a coefficient of determination, $R^2 = 0.99$. The curve for model D (random sequence) divides the *x*-*y* space into two regions: clustered sequences plot above it while anticlustered ones plot below. For example, while the curve for model A (uniformly spaced fractures) lies below that of model D, models B and C (evenly spaced fracture clusters) lie above it.

While the lacunarity curves as described above can delineate overall differences between the models, the subtle breaks in these curves that potentially correspond to major breaks within the sequences are not easy to locate. Hence, it is important to consider their first-order derivatives that amplify such breaks and can identify finer details of the scale-dependent patterns in the scan lines (Figure 3). The first-order derivative curve for the uniform model A (Figure 3a) breaks at $r = 22$ units that exactly matches the constant fracture spacing value and thereafter oscillates about the homogenous line (slope = 0) denoting that the sequence is a uniform one throughout its entire length. The curve for model B has two main slope breaks: the first abrupt jump at $r = 8$ and the largest trough at $r = 161$ (Figure 3b). The first break is equal to the fracture spacing within the clusters while the second break closely approximates the intercluster spacing of 162 units.

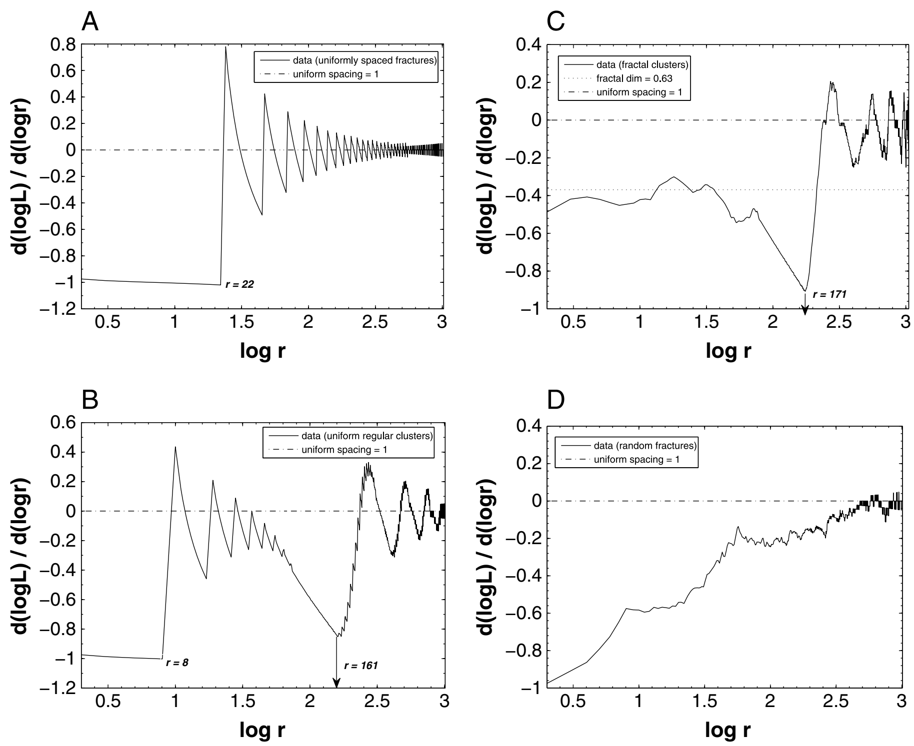

Figure 3. Lacunarity slopes for models A, B, C, and D. Breaks in slope correspond to spacings at given scales, e.g., model B, breaks at log $r \sim 0.9$ ($r = 8$) and log $r \sim 1.35$ ($r = 22$) denoting spacing within clusters and spacing between clusters, respectively.

Model C breaks at $r = 171$ which is a large trough as seen in Figure 3c. This value matches with the first intercluster spacing which is 172 units. To the left of this trough the curve is subparallel to the x axis with only minor peaks and troughs about a line that represents the known fractal dimension of 0.63 for the Cantor bar used in modeling the clusters. At scales larger than $r = 171$, the sequence oscillates about the homogenous line (slope ~ 0) like in model B indicating that at these scales (larger than the intercluster distance) the sequence is a uniform one. This happens because the fractal clusters are evenly spaced along the scan line. Model D being random does not show any specific trend (Figure 3d).

All of the fracture organization parameters extracted from the lacunarity analyses of the synthetic scan lines are summarized in Table 1. It can be concluded from our models that lacunarity curves and their slopes can delineate scale-dependent pattern changes within the same scan line as well as the scales at which these changes take place. Patterns that appear clustered at one scale and fractal or uniform over another can be identified.

4. Application to Natural Data

The lacunarity derivative curve for detecting changes in spatial clustering with scale as developed in the last section was applied to two scan line data sets collected from the Monterrey salient, Sierra Madre Oriental, NE Mexico, by *Gomez* [2007]. These fracture data constitute veins in carbonate layers of the Lower Cretaceous Cupido Formation. The data were obtained from layers 11 and 13 in the Palmas canyon and are henceforth referred to as P11 and P13, respectively (Figures 4a and 4b). The former is a 21 m long scan line with fourteen 220 mm wide fractal clusters spaced at about 1.1 m. The clusters have a fractal dimension of 0.42 [*Gomez*, 2007, Table 7.2 therein]. P13 is a 5.5 m long scan line with randomly arranged fractures [*Gomez*, 2007]. The data were discretized on the millimeter scale following the scheme of *Priest and Hudson* [1976]. A unit length is 1 mm such that 1 mm spacing is represented by a 0 and a fracture by 1 thus yielding a sequence of zeros and ones, essentially, a one-dimensional binary data set. The coefficient of variation parameter [*Gillespie et al.*, 1999] yielded a value of $C_v = 1.7$ for P11 which indicates a clustered sequence. For P13, $C_v = 0.9$ indicating a near random arrangement.

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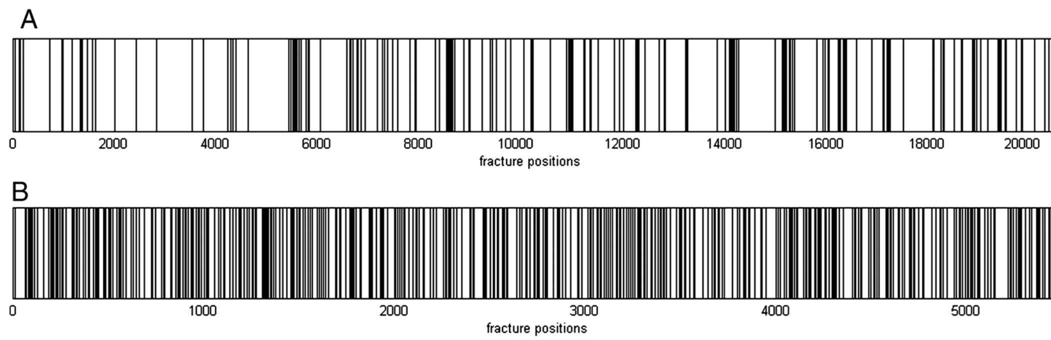

Figure 4. Natural scan lines from *Gomez* [2007] where fractures are shown by lines: (a) P11 = fractal clusters and (b) P13 = random fractures. Note the differences in fracture intensity (ϕ).

Figure 5 shows the lacunarity curves of P11 and P13. The former has 257 recorded fractures along a 21 m line (fracture intensity, $\phi = 0.012$), while the latter has 459 fractures along a 5.5 m line ($\phi = 0.083$). This difference in fracture intensities (which is clearly apparent visually in Figure 4) leads to the offset in L values at $r = 1$ given by $1/\phi$, such that log L (1) of P11 is 1.92 and that of P13 is 1.08. The straight line segment of the P11 lacunarity curve indicates a constant slope and as seen from equation (3b). This behavior implies a fractal organization over the scale of the segment in question. This is comparable to model C (Figure 2) which is composed of fracture clusters with fractal organization within. It may be noted here that the coefficient of variation values for P11 ($C_v = 1.7$) and model C ($C_v = 2.43$) is very different; clearly, this parameter fails to recognize the similarity. P13 has a concave-up lacunarity curve similar to that for the random population of fractures in model D (Figure 2). This is consistent with the findings of *Gomez* [2007] that P13 is populated by randomly distributed fractures.

Figure 6a is the first-order derivative of the lacunarity curve of P11 and is comparable to that of model C in the previous section. Between scales of $r \sim 25$ mm and 740 mm (i.e., log $r \sim 1.4$ and 2.87) the pattern shows a relatively flat line thus indicating a fractal organization. These bounds are similar to the bounds of 12 mm and 664 mm found by *Gomez* [2007]. The fractal dimension, D, of fractures within the clusters was found from fitting a linear model to this segment between $r \sim 25$ mm and 740 mm in the log $L(r)$ versus log r curve that yielded an $R^2 = 0.99$ and applying equation (3b). The resulting value of $D \sim 0.52$ is slightly higher than the value of $D = 0.44$ found by *Gomez* [2007]. Beyond $r \sim 740$ mm the sequence starts to approach uniform behavior indicating that the clusters themselves are spaced at regular intervals just as in model C. A major difference being that P11 has fractures in the intercluster regions. This is the reason that the transition from fractal to uniform behavior is more continuous, and there is no distinct trough as seen in Figure 3c. The different fracture organization parameters extracted by lacunarity analyses of the P11 scan line data are summarized in Table 1.

Figure 6b is the first-order derivative of the lacunarity curve of P13 and is very unlike those of models A, B, or C in Figure 3. Also, it has no sharp breaks in its slope indicating that it behaves the same way at all scales. Although there seems to be some differences between P13 and the random model D in terms of their lacunarity derivative curves, their lacunarity curves look similar as discussed in the last paragraph and it may therefore be concluded that P13 is indistinguishable from a random distribution at all scales.

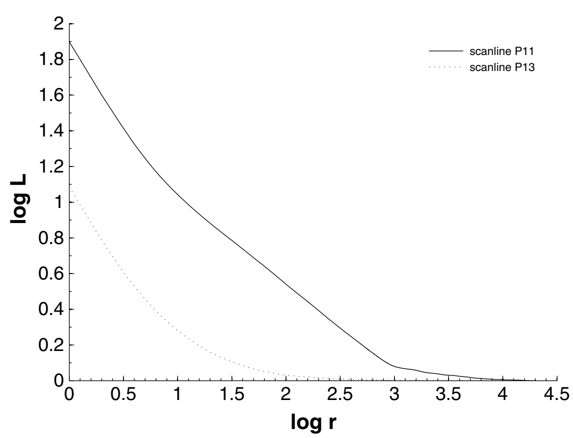

Figure 5. Lacunarity curves for natural scan lines P11 and P13. The two are offset because P11 has a lower intensity, hence higher lacunarity than P13.

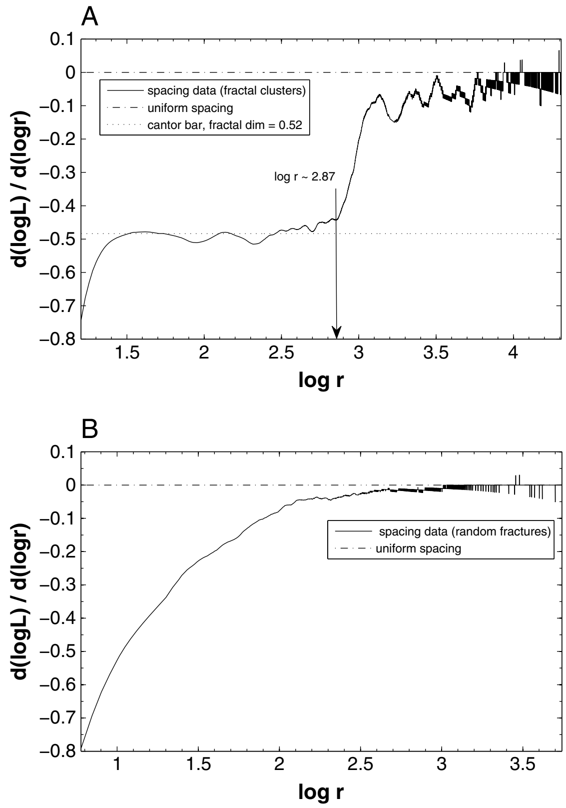

Figure 6. Lacunarity slopes for natural data. (a) P11 showing fractal ($D = 0.52$) and uniform behavior at different scales and (b) P13 indistinguishable from random.

5. Concluding Remarks

The first-order derivative of a lacunarity curve can be used to detect breaks in the slope of the lacunarity curve in order to find scales at which a pattern changes its spatial distribution. For clustered populations, plotting the slope against the scale can reveal the intercluster spacing and possible fractal or random organization within the clusters. Further, where fractures within a cluster are periodically spaced, the spacing at that scale can also be found as with model B. This kind of analysis is thus important where in addition to an average spacing value, more information on the spatial distribution of fractures may be desired. For example, natural scan line P11 has an average fracture spacing of 78 mm, but the fractures are actually present in clusters that are uniformly spaced when observed at scales larger than $r = 740$ mm.

The technique described here can be extended along various avenues. The transition from one-dimensional analysis of fractures to that of fracture networks in two dimensions and three-dimensional volumes is an obvious area for further exploration. *Roy et al.* [2010] analyzed two-dimensional fracture networks for lacunarity in order to delineate differences between maps with similar fractal dimensions but different clustering attributes. In principle, the same algorithm can be implemented to create curves representing log-transformed lacunarity and lacunarity slopes, as in Figures 5 and 6, respectively, that would delineate scale-dependent heterogeneity in two- and three-dimensional fracture patterns just as they do in the case of the one-dimensional data sets described in this research. Further, lacunarity can be used in conjunction with map counting [*Kruhl*, 2013] by replacing the fractal dimension with a single lacunarity number [*Roy and Perfect*, 2012], thus creating a contour map depicting regions that have higher clustering of fractures.

Our present technique, like most others, considers only fracture spacing in determining scale-dependent heterogeneity. It does not include a consideration of the individual fractures in terms of parameters such as aperture, length, and dip. As per the cubic law [*Snow*, 1969], fracture width controls flow within a system. Also, it is the widest fractures that accommodate most of the strain. Apart from fracture width, it would also be interesting to investigate if long fractures or steeply dipping ones occur within clusters found in fault damage zones. Therefore, a further improvement to this new technique would be to include data on fracture widths, lengths, or dips together with spacing in order to test if large or steeply dipping fractures occur in clusters.

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